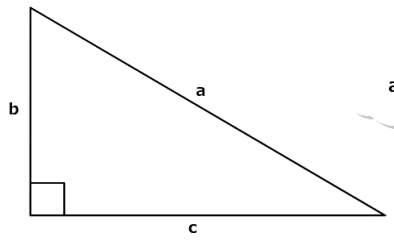
Notes about trigonometry for UEENEEH114A Resonance Circuits – Greg Moore

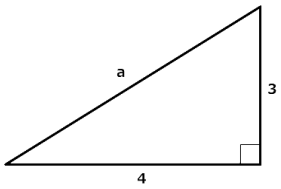
Here are some notes about Pythagorean theorem and later some trigonometry leading towards Cartesian to Polar conversions. You need to know this for the phasors involved with AC complex angles in this unit.

The square on the hypotenuse is equal to the sum of the squares on the other two sides



$a^2 = b^2 + c^2$

From Pythagoras, $a^2 = b^2 + c^2$



Substitute for b and c,

$a^2 = 3^2 + 4^2$

$a^2 = 9 + 16$

$a^2 = 25$

Square root both sides,

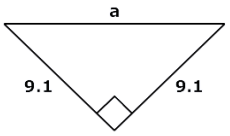
$a = \sqrt{25}$

$a = 5$

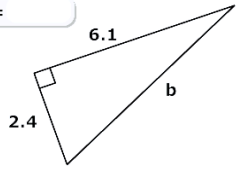
Write the answers to the following hypotenuse dimension to one decimal place.

Calculate the hypotenuse.

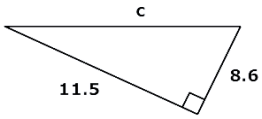
1) a =



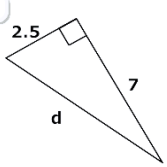
2) b =



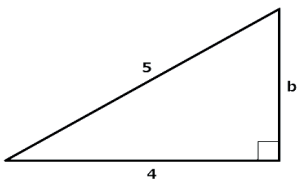
3) c =



4) d =



From Pythagoras, $b^2 + c^2 = a^2$



Substitute for a and c,

$b^2 + 4^2 = 5^2$

$b^2 + 16 = 25$

$b^2 = 25 - 16$

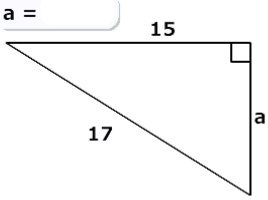
$b^2 = 9$

$b = \sqrt{9}$

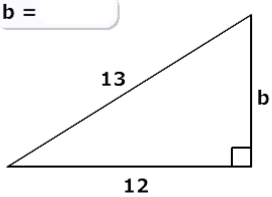
Write the answers to the following triangle side dimension.

Calculate the missing side in these right-angled triangles.

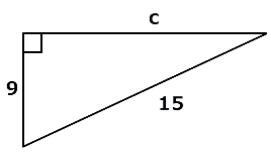
1) a =



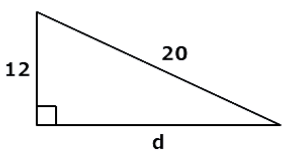
2) b =



3) c =



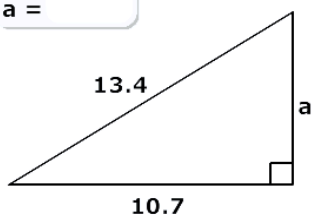
4) d =



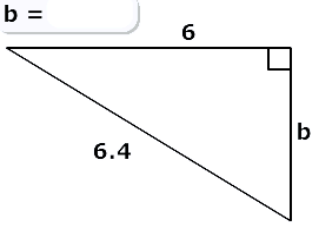
Solve the missing side to one decimal place.

Calculate the missing sides.

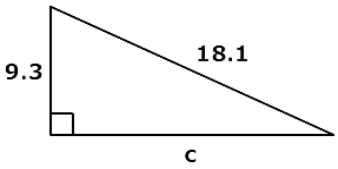
1) a =



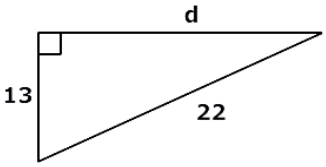
2) b =



3) c =



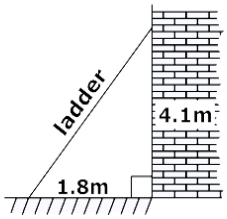
4) d =



Answer the questions.

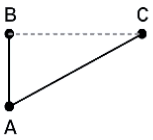
1) **The bottom of a ladder rests 1.8m from the base of a wall. The top of the ladder rests against the wall at a height of 4.1m. What is the length of the ladder to the nearest 10cm?**

m



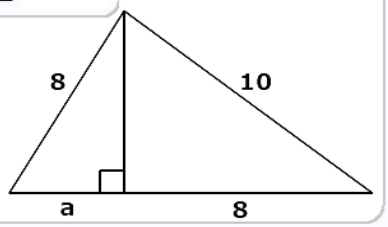
2) **A is 55km South of B. C is East of B and 128km from A. How far is it from B to C, to the nearest kilometre?**

km

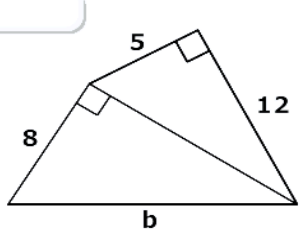


A little bit more thought needed here!

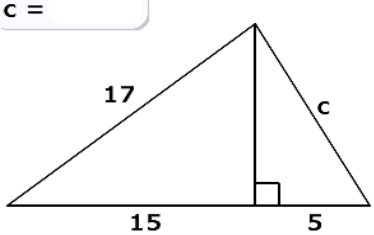
1) a =



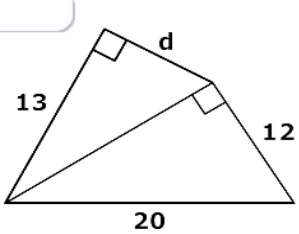
2) b =



3) c =



4) d =



Answer the questions.

1) A square has a perimeter of 36cm. What is the length of its diagonal?

cm

2) Noni walks 390m North, then 280m East. How far is she from the starting point?

m

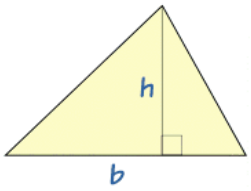
3) The diagonal of a square is 16cm long. What is its perimeter?

cm

4) A square has an area of 144cm^2. What is the length of its diagonal?

cm

Area



The area is **half of the base times height**.

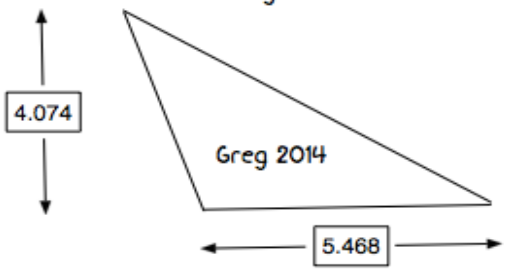
- "b" is the distance along the base
- "h" is the height (measured at right angles to the base)

Area = ½ × b × h

The formula works for all triangles.

Note: another way of writing the formula is **bh/2**

Example, what is the area of this triangle?

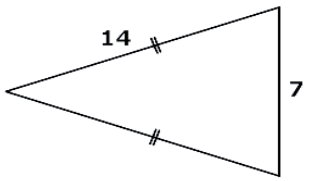


Example, Half the base dimension X the height.

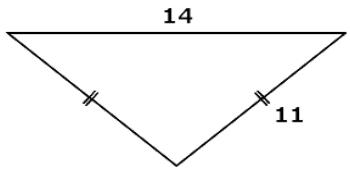
$$A = (\frac{5.47}{2} \cdot 4.07) = A = 11.1$$

Calculate the area of the Isosceles triangles.

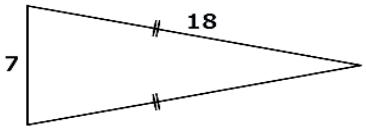
1) Area = cm^2



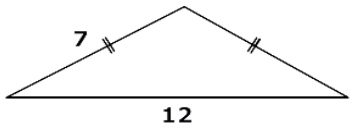
2) Area = cm^2



3) Area = cm^2

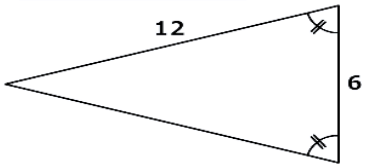


4) Area = cm^2

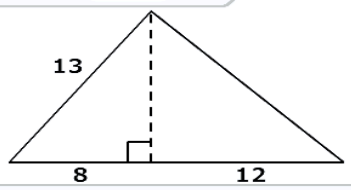


Calculate the area of the shapes.

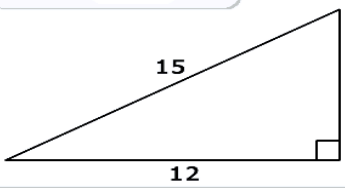
1) Area = cm^2



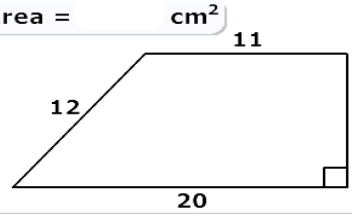
2) Area = cm^2

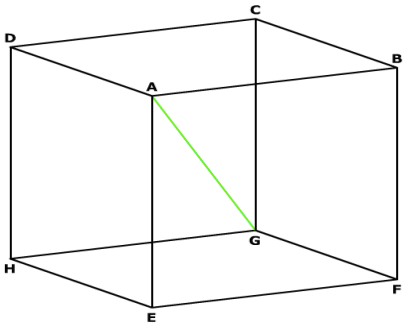


3) Area = cm^2



4) Area = cm^2

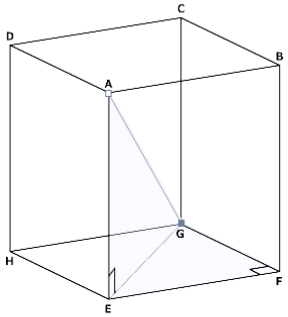




What is the length of the diagonal of a cube with side length of 5cm?

Study the explanation here and then attempt the questions over page.

From triangle EFG, find EG. Then from triangle AEG, find AG.



In $\triangle EFG$,

$EG^2 = 5^2 + 5^2$

$EG^2 = 25 + 25$

$\underline{EG^2 = 50}$

In $\triangle AEG$,

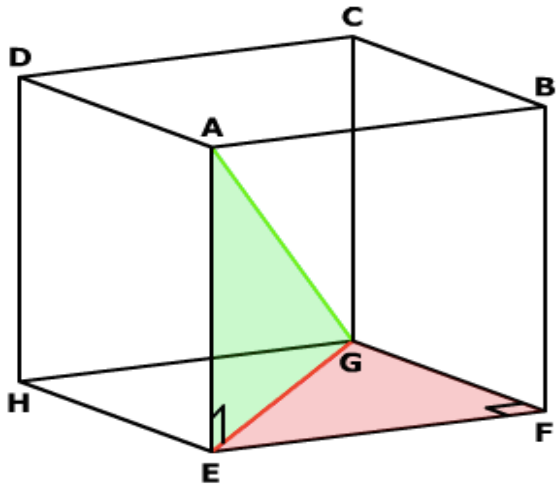
$AG^2 = EG^2 + 5^2$

$AG^2 = 50 + 25$

$AG^2 = 75$

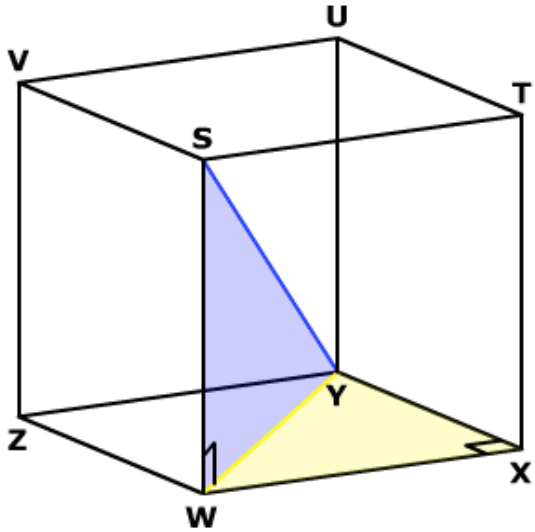
$AG = \sqrt{75}$

$\underline{AG = 8.7cm}$ (to 1 dp)



Find the length of the diagonal A – G.

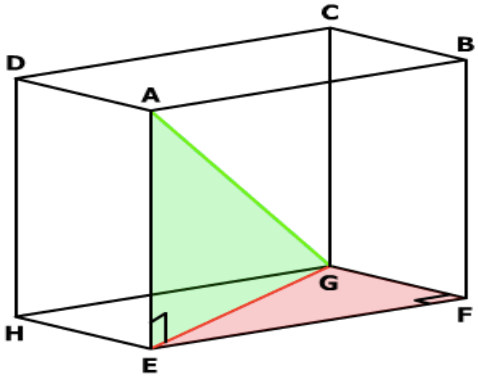
Side length = 6cm



Find the length of the diagonal S – Y.

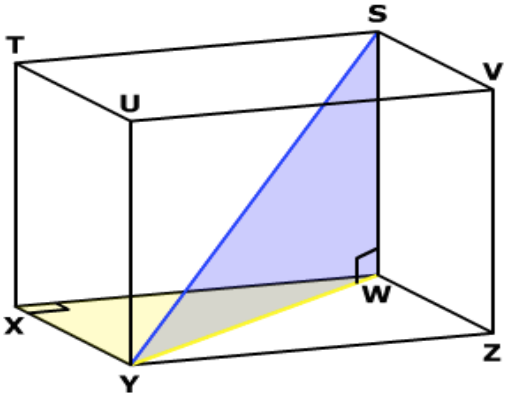
Side length is 8cm

Calculate the diagonal of each cuboid.

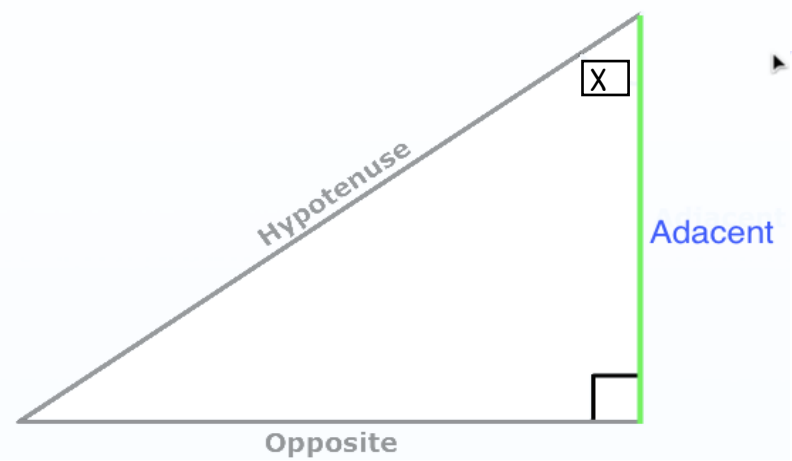


Cuboid dimensions: 5X10X16
Find diagonal A – G

Cuboid dimensions: 6X7X8
Find diagonal S – Y



In trigonometry we use names Hypotenuse, Adjacent and Opposite to name and identify sides on a right angled triangle.



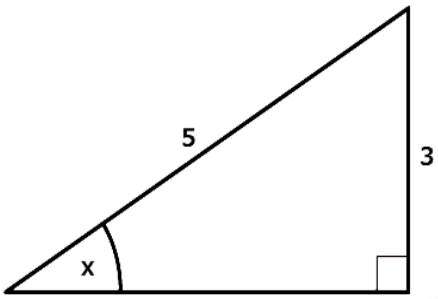
$$\text{Sin } x = \frac{\text{Opposite}}{\text{Hypotenuse}}$$

$$\text{Cos } x = \frac{\text{Adjacent}}{\text{Hypotenuse}}$$

$$\text{Tan } x = \frac{\text{Opposite}}{\text{Adjacent}}$$

To calculate an angle using sin...

From SOHCAHTOA, $\sin x = \frac{\text{opp}}{\text{hyp}}$

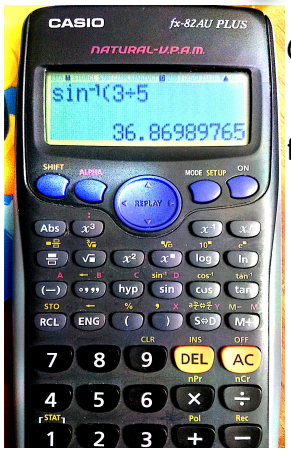


Substitute for opp and hyp,

$$\sin x = \frac{3}{5}$$

$$x = \sin^{-1}\left(\frac{3}{5}\right)$$

$$x = 36.9^{\circ} \text{ (to 1 dp)}$$

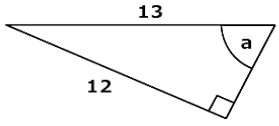


Greg's trusty fx-82

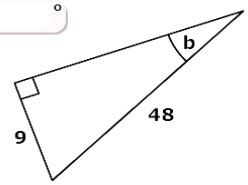
In electronics study of AC circuits we commonly use the Sin function for calculations.

Calculate the required angle using sin.

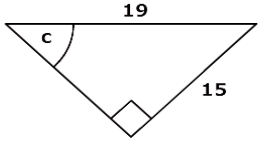
1) a = °



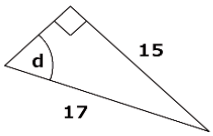
2) b = °



3) c = °

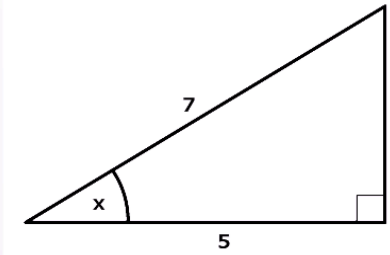


4) d = °



To calculate an angle using cos...

From SOHCAHTOA, $\cos x = \frac{adj}{hyp}$



Substitute for adj and hyp,

$$\cos x = \frac{5}{7}$$

$$x = \cos^{-1}\left(\frac{5}{7}\right)$$

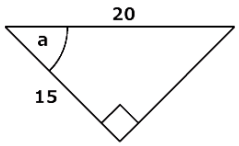
$$x = 44.4^{\circ} \text{ (to 1 dp)}$$

Try to do these COS problems. When doing cartesian to polar calculations you will need to use COS. And there is the Cosine rule you will learn.

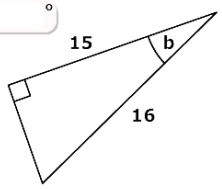
In fact a good working knowledge of all the three trigonometry functions is important.

Calculate the required angle using cos.

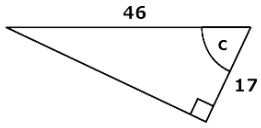
1) a = °



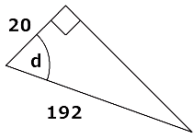
2) b = °



3) c = °



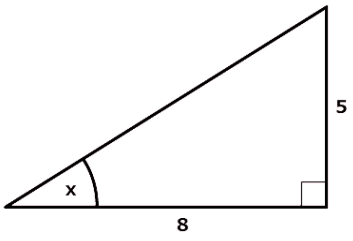
4) d = °



To calculate an angle using tan...

From SOHCAHTOA, $\tan x = \frac{opp}{adj}$

toa



Substitute for opp and adj,

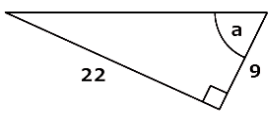
$$\tan x = \frac{5}{8}$$

$$x = \tan^{-1}\left(\frac{5}{8}\right)$$

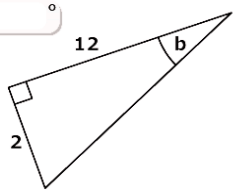
$$x = 32.0^{\circ} \text{ (to 1 dp)}$$

Calculate the required angle using tan.

1) a = °

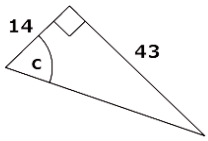


2) b = °

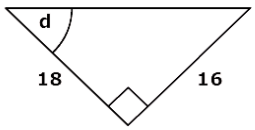


Some calculations using TAN Write the answers for each of the angles.

3) c = °

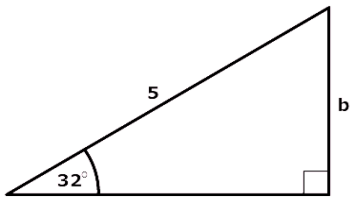


4) d = °



To calculate a side using sin...

From SOHCAHTOA, $\sin x = \frac{opp}{hyp}$



Rewrite,

$\frac{opp}{hyp} = \sin x$

Substitute,

$\frac{b}{5} = \sin 32$

Multiply both sides by 5,

$b = 5 \times \sin 32$

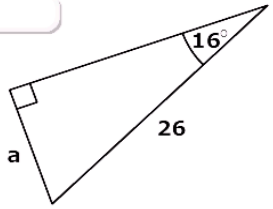
$b = 2.6$ (to 1 dp)

To find an unknown side length, either of SIN or COS or TAN will be used depending on which sides and angles you know.

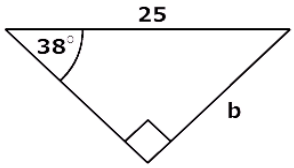
Calculate the side as identified using SIN

Use sin to calculate the required side.

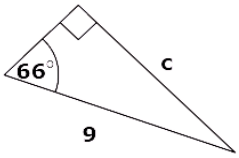
1) a =



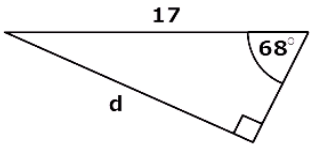
2) b =



3) c =

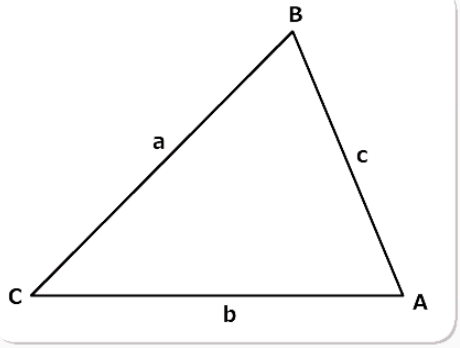


4) d =



For Non-Right angled triangles, you can use the Sin rule or the Cosine Rule.

The cosine rule allows you to calculate sides and angles in non right-angled triangles.



To calculate a side we start with this form...

$a^2 = b^2 + c^2 - 2bc \cos A$

To calculate an angle we start with this form...

$\cos A = \frac{b^2 + c^2 - a^2}{2bc}$

To calculate a side, we need the other 2 sides and the angle between them.

To calculate an angle, we need all 3 sides.

If you look carefully at the law of cosines, you should see a resemblance to the Pythagorean theorem. In fact, for right triangles, the law of cosines simplifies to the Pythagorean theorem. Try it yourself. The last term drops out (since cos 90 = 0) and you're left with the familiar formula of c2 = a2 + b2.

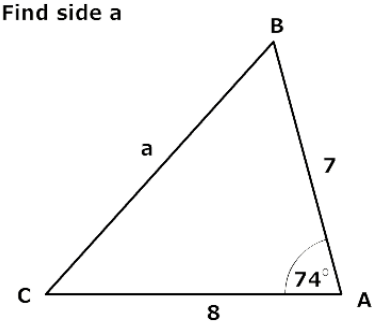
If you're curious, the 2ab cos(C) term compensates for the lack of a right angle.
The law of cosines allows you to solve any triangle for which you know any three of the four unknowns in the formula.

A pretty solid rule and able to solve many problems.

To calculate a side using the cosine rule...

From the cosine rule, $a^2 = b^2 + c^2 - 2bc\cos A$

Find side a



Substitute into above formula,

$a^2 = 8^2 + 7^2 - 2\times8\times7\times\cos 74$

$a^2 = 64 + 49 - 112\times\cos 74$

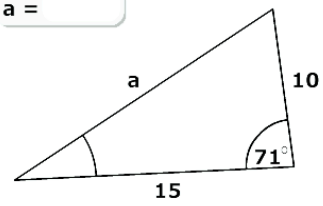
$a^2 = 82.12861615$

Square root both sides,

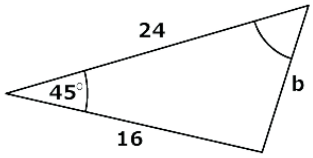
$a = 9.1$ (to 1 dp)

Calculate the required sides.

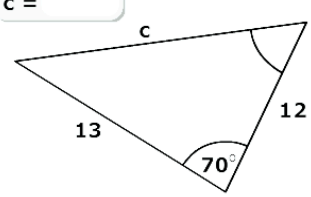
1) a =



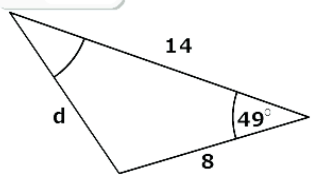
2) b =



3) c =



4) d =

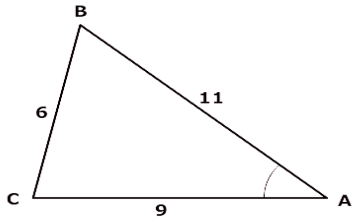


Use the cosine rule to solve the missing side.

To calculate an angle using the cosine rule...

From the cosine rule, $\cos A = \frac{b^2 + c^2 - a^2}{2bc}$

Find angle A



Substitute into above formula,

$\cos A = \frac{9^2 + 11^2 - 6^2}{2\times9\times11}$

$\cos A = \frac{166}{198}$

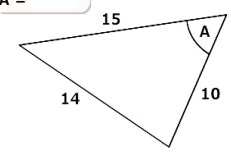
$\cos A = \frac{83}{99}$

$A = \cos^{-1}\left(\frac{83}{99}\right)$

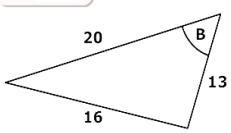
$A = 33.0^\circ$ (to 1 dp)

Calculate the required angles.

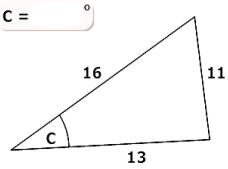
1) A = °



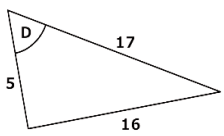
2) B = °



3) C = °



4) D = °



Try to solve the angles A, B, C, D by using the cosine rule.

I have covered most of what you need to know in basic trigonometry here and I have a second handout about Cartesian to Polar conversion.

GM