Week 7 – Single Index Model & Mispricing's

The Single Index Model

We cannot observe the true market portfolio – we pick something similar such as a broad value-weighted index *e.g. S&P500.* The empirical model is called the SIM *(since not true market portfolio – application to real world).* We are interested in variations in the cross section & over time *(add time indices)* & we only see realised returns *(not expected).*

$r_{it} - r_{ft} = \alpha_i + \beta_i(r_{Mt} - r_{ft}) + \varepsilon_{it} \rightarrow allow\ the\ intercept\ to\ not\ be\ the\ risk\ free\ rate$

SIM excess return of a stock made up of;

- α – constant, it is the *expected excess return when the excess return on the market is 0*. Distinguishes securities
- $\beta_i(r_{Mt} - r_{ft})$- a variable component *(related to market performance – common macro factor)*
  Stocks risk premium = market risk premium * sensitivity *(systematic)* & alpha *(non-market premium (**S.A**))*
- Ɛ - random residual component *(no correlations & 0 expected value).* It refers to firm specific events

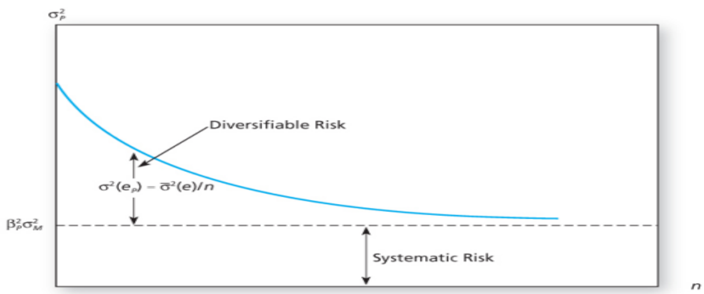
SIM & Portfolio Management (Benefit of SIM vs. MPT)

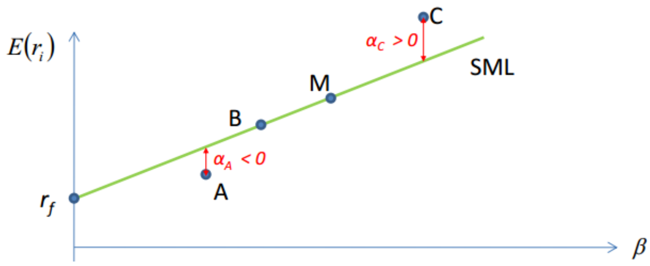
Markowitz's portfolio optimisation process with 'n' stocks requires 'n' expected returns, variances & (n^2 – n)/2 covariance's. If n =100, we need 5150 covariance's. But, we know that $\sigma_i{}^2 = \beta_i^2\sigma_m^2 + \sigma_\varepsilon^2 \rightarrow$ $Cov(r_i, r_j) = \beta_i\beta_j\sigma^2{}_M$ → Now we only need 302 estimates *(3n + 2) – big advantage of the SIM (all asset returns derived from common factor →* r_M*).*

SIM & Diversification

$Portfolio\ Risk = \sigma_P{}^2 = \beta_i^2\sigma_m^2 + \sigma^2(\varepsilon_P) \rightarrow where\ \sigma^2(\varepsilon_P)\ depends\ on\ \varepsilon_i(firm\ specific\ components).$

$Since\ the\ \varepsilon_i\ are\ uncorrelated\ \sigma^2(\varepsilon_P) = \sum_{i=1}^{n}\left(\frac{1}{n}\right)^2\sigma^2(\varepsilon_i) = \frac{1}{n}\sigma^{-2}(e)$ → As n gets large $\sigma^2(\varepsilon_P) \rightarrow 0$. Firm specific risk is diversified away.





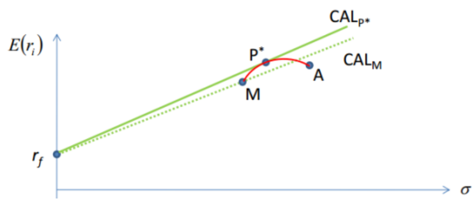
The Optimal Risky Portfolio = Active (Security Analysis) + Passive (Aid Diversification)

Jensen's Alpha

According to the CAPM, α should = 0. If it is not, the asset is mispriced *(or our model is wrong)* & α ≠ 0 *(α measures mispricing)*. In the graph, A & C are mispriced & can be exploited by tilting our portfolio weights away from the market portfolio. Thus P*≠M.

- Combine an 'active position' w_A in the mispriced asset with $1 - w_A$in the market portfolio
  - $r_p = w_A r_A + (1 - w_A)r_M \&\ try\ maximise\ the\ Sharpe\ ratio\ as\ before \rightarrow \underset{w_A}{Max}\ S_p = \frac{E(r_p[w_A]) - r_f}{\sigma_P[w_A]}$
  - $Exploiting\ 'A'\ allows\ us\ to\ create\ a\ new\ efficient\ frontier, a\ new\ optimal\ risky\ portfolio\ \&\ a\ new\ CAL$

Exploiting Mispriced Assets – Active Portfolio Management



A consequence of taking an active position *(to exploit α)* we incur unpriced systematic risk – we need to weight costs *(unsystematic risk)* & benefits (*mispricing).*

$$Optimal\ Mispriced\ Asset\ Weight = w_A = \frac{w_A^0}{1 + w_A^0(1-\beta_A)} \ \&\ W_A^0 = \frac{\frac{\alpha_A}{\sigma^2(e_A)}}{\frac{E(R_M) - r_f}{\sigma_M^2}}$$

W_A^0 *is the extra return per unit of risk we get from our active portfolio in relation to the return per unit of risk we get from the market port*

w_A *is the adjustment to take the* β (*not* 1)*of the active asset/portfolio into account & is the weight in the mispriced asset portfolio*

Several Mispriced Assets

If we uncover a few mispriced assets at the same time we form an active portfolio, the sum of the weights add up to w_A.

Each mispriced asset is weighted in the active portfolio according to its mispricing to unsystematic risk ratio $w_i^0 = \frac{\frac{\alpha_i}{\sigma^2(e_i)}}{\Sigma\frac{\alpha_i}{\sigma^2(e_i)}}$ & the contribution to the information ratio of the active portfolio $[\frac{\alpha_A}{\sigma^2(e_A)}]^2 = \sum(\frac{\alpha_i}{\sigma^2(e_i)})^2$. We treat the active portfolio as one asset & the analysis from there is exactly the same as the 1 mispriced asset scenario. The more diversified the active portfolio, the greater the weight.

Optimisation Summary

1. Compute weights of each security in the active portfolio *(scale according to mispricing to unsystematic variance ratio)*
2. Compute the alpha of the active portfolio *(weighted average)*
3. Compute residual variance of active portfolio → $\sum w_i^2 \sigma^2(\varepsilon_i)$
4. Compute initial position of active portfolio → $\frac{\alpha_A}{\sigma^2(\varepsilon)_i} / \frac{E(r_m) - r_f}{\sigma_m{}^2}$
5. Compute Beta of active portfolio *(weighted average)*
6. Adjust initial position in active portfolio → $\frac{w_A^0}{1 + w_A^0(1-\beta_A)}$

Gains from Exploiting Mispriced Assets

There is a direct relationship between an assets appraisal/information ratio $\frac{\alpha_A}{\sigma(e_A)}$ & the improvement in the Sharpe Ratio.

$$S_P^2 = S_M^2 + \left(\frac{\alpha_A}{\sigma^2(e_A)}\right)^2$$

Arbitrage Pricing Theory (APT)

APT proposed by Stephen Ross *(1976)* predicts a SML linking E(r) & risk *(differs to CAPM SML).* It applies to well diversified portfolios *(unsystematic risk = 0, B=1)* which must lie on SML *(risk premiums proportional to beta).* Relies on 3 assumptions;

- Security **returns** can be **described by** a **factor model**
- **Sufficient securities** to **diversify** away **unsystematic risk**
- Functioning **markets don't allow** for the **persistence of arbitrage opportunities (no arbitrage condition)**

Factor Models

In the CAPM, we can refer to the market portfolio as a risk factor, whilst the β is the loading on that factor. In reality, there may be other risk factors with which assets that co-vary *(load) lea*ding to higher required returns for those assets. While this is theoretically different from CAPM's argument, the interpretations are similar. Each new risk factor is self-financing.

$$E(R_p) = R_f + \beta_1 E(Risk\ Factor\ 1) + \beta_2 E(Risk\ Factor\ 2) + \cdots + \beta_n E(Risk\ Factor\ n) \rightarrow \beta\ calculated\ by\ COV\ with\ return$$
$$\rightarrow called\ loading\ zones\ (sensitivity) \rightarrow COV(RF1, RF2) = 0, each\ \beta RF\ is\ the\ risk\ premium\ attributed\ to\ 'x'$$

Assuming we know the factors that can be used to model security returns *(e.g. GDP growth)*, only factor risk is priced, otherwise there would be arbitrage opportunities *(also nonfactor risk can be diversified away).* Hence, only returns for well diversified portfolios can be modelled by these factors – *express the returns of stock using factor portfolios which represent an economic risk for the stock market*. However:

- ✓ A small number of mispriced individual stocks is possible *(cannot rule out violations of equilibrium for any asset)*
  - If many stocks violated the expected return/risk r/ships then the diversified portfolio would violate APT
    - The expected return, beta r/ship would not hold *(excess returns must be proportional to beta's)*

A ***factor portfolio*** is a well-diversified portfolio *(unsystematic risk = 0)* with β=1 on 1 of the factors & β=0 on the other factors. It tracks a particular source of macro risk *(uncorrelated with the other risks).*

Problems with APT → does not provide appropriate guidance on factors → choose important risk factors

APT vs. CAPM

- APT is more practical → CAPM based on unobservable market portfolio, APT benchmark portfolio just needs to be a well-diversified portfolio with a beta of 1
- CAPM relies on mean variance efficiency
  - CAPM → Actions of many small investors restore equilibrium *(risk return dominance)*
  - APT → actions of a few investors can eliminate arbitrage opportunities & restore equilibrium *(no arbitrage)*
- CAPM requires equilibrium for all assets, APT cannot rule out violations of equilibrium for any individual asset
  - CAPM provides a statement on the E(r), beta r/ship for all securities. APT – most securities
- APT Breaks down Systematic Risk into Components *(various macroeconomic variables)*.

Factor Portfolio Example – Fama & French (1996)

$$E(r_i) = r_f + \beta_i^M[E(r_M) - r_f] + \beta_i^{SMB}(r_{SMB}) + \beta_i^{HML}(r_{HML}) + \beta_i^{MOM}(r_{MOM}) \rightarrow loading\ of\ any\ asset\ depends\ on\ its\ COV\ with\ the\ factor$$

SMB – Small (long30%) minus Big (short 30%), HML – High (book to market – 50%) minus Low (short bottom 50%), MOM – momentum (30% in preceding 'winners' + short the worst 30% performing stocks) → All self-financing (0 net cost factor portfolios).

Empirical Results – asset loadings on these factors tend to predict expected returns *(violating CAPM) – but we don't know why.* Additionally, it is not clear why these factors are 'risk factors' *(what risks are associated with them?).* It is possible the results are because of data snooping. Although it is based on historical data, it is used more than CAPM in practice.

Week 8 – EMH & Behavioural Finance

Efficient Market Hypothesis

Random Walk

If you believed a stock was going to rise 10%, you would buy it immediately. But if everyone has access to the same information no one would sell & the price would rise immediately – a forecast about favourable future performance = favourable current performance. ASSET PRICES SHOULD REFLECT ALL AVAILABLE INFORMATION. Because new information *(price driver)* is unpredictable, price changes are unpredictable & should follow a 'random walk' *i.e. price changes are random & unpredictable (price levels are predictable given new information).*

EMH –Efficiency with respect to information efficiency (how quickly & accurately information is incorporated into prices)

The asset MGMT industry is competitive. Strong incentives for outperformance increases effort on research – with so many people looking for new info, the chances of finding it are slim – cannot abnormally profit on information already known.

- **Weak** – current prices reflects information from historical trading data – *past prices, volumes & short interest*
  - Implies: technical analysis cannot earn abnormal returns *(doesn't work)*