Week 1 – Introduction & Bond Pricing

Bond Pricing

A bond is a claim on some fixed future cash flow(s). It matures at time 'T' when the last CF is paid – usually large, consisting of the face value. Regular intermittent CF's are coupons *(generally expressed as a % of the par value – coupon rate)* but some bonds do not have these *(zero coupon bonds).*

Default Risk is the risk that you will be unable to collect your CF's from the counterparty *(promises don't always deliver).* It's important in practice, but **ignored** in this course. Other frequent ***assumptions*** include **no transaction costs, constant interest rates, complete markets**.

Approaches to Pricing

1. ***Fundamental pricing*** – prices are set in a demand-supply equilibrium
   - The properties of an asset tell us what the price is likely to be
   - Used later in the course
2. ***Arbitrage pricing*** – involves taking some price as given & pricing other assets relative to that
   - Used for pricing bonds & derivatives

Arbitrage

An arbitrage is a set of trades that delivers a positive risk free cash flow today whilst generating zero cash flows in the future – 'free lunch' or 'money machine'. *Arbitrage example: if an identical bond were selling for 2 different prices (violating the law of 1 price), one could buy the cheap bond & short the expensive bond.* All arbitrages are based on this principle but the trades may be more complex.

Replicating Portfolio's (Synthetic Assets)

For arbitrage, we rely on a portfolio of assets that exactly replicate the CF's of another asset. We ensure we construct the portfolio using assets with known prices.

Pricing a Zero Coupon Bond

YTM *(rate earned on a similar risk investment)* = 10%, FV = $100, T = 1. We can lend/borrow from a bank @ 10%

$$\mathrm{P} = \frac{100}{1.1} = 90.9 \rightarrow \text{but what is the economic logic?}$$

Suppose Bond A trades at $80.90

1. **Replicate the Asset** – create the same cash flows – we need to receive $100 in 1 years' time
   a. Deposit an amount of money, M into the bank
2. At time T=1 we receive 1.1M which should equal = $100 *(law of one price)*
   a. *1.1M = 100*, M = 100/1.1 = 90.9

Exploiting the Mispricing

- Buy the cheap instrument *(Bond A)*, sell the expensive instrument *(synthetic asset i.e. borrowing money)*
  - Borrow $90.90, Buy the bond for $80.90 → $10 profit today
  - In one year the bond pays us $100 which is the exact amount to repay the loan → $0 net cash flow
- Our 'free' $10 in arbitrage profit & the entire scheme is an arbitrage trade

[Insert Time Line]

Arbitrage Pricing

In reality, intelligent investors locate arbitrage opportunities & trade on them. In the previous example, this would see an increase in demand for the bond, causing an increase in price until the equilibrium – arbitrage free price.

We cannot say whether the bond price or the banks interest rate is wrong – we can only prove (& care) if the prices are internally consistent.

Pricing a Coupon Bond

FV = $100, C = 5%, Y1 = y1, Y2 = y2, T=2

1. **Replicate the entire CF stream we want to price *(Strategy)***
   a. For there to be no arbitrage, the price of the CF stream must be the same as the price of the replication.
   b. Treat it as 2 zero coupon bonds & replicate via 2 deposits
2. **At T = 1, we want to receive $5**
   a. Deposit M1 → = M1*(1+y1) at T = 1 → M1 = 5/(1+y1)
3. **At T = 2, we want to receive $105**
   a. Deposit M2 *(at t = 0)* → M2(1+y)^2 at T = 2 → M2 = 105/(1+y2)^2
4. **Law of One Price – cost of bond = cost of replication = M1 + M2 = 5/(1+y1) + 105/(1+y2)^2**

Pricing Formula & YTM

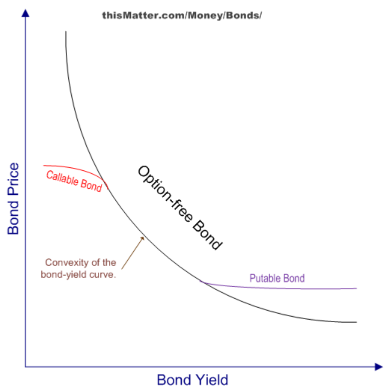
With many CF's, discounting each one gets tedious → compact pricing formula. To do this, assume constant interest rates.

Perpetuities

Perpetuity is a never ending constant cash flow stream. To replicate this we deposit $M into a bank account & withdraw interest every year. *How large should M be to give interest of C? My=C, M = C/y.*

What Do We Need to Replicate – a coupon stream of 'c' from 1 to t & a large payment of FV at t.

1. **FV Payment @ t**
   - $PV = \frac{FV}{(1+y)^t}$t
2. **Coupon Stream**
   - Replicated by the difference between a perpetuity with 1st payment at t=1 *(X1)* & and another at t = T+1 *(X2)*
     i. PV(Cs) = PV(X1) – PV(X2) = $\frac{c}{y} - \frac{\frac{c}{y}}{(1+y)^t} = \frac{c}{y}\left[1 - \frac{1}{(1+y)^t}\right]$



$$\therefore P = \frac{c}{y}\left[1 - \frac{1}{(1+y)^t}\right] + \frac{FV}{(1+y)^t}$$

YTM & Bond Prices

As the **YTM increases, bond prices fall**. Additionally, the price at higher YTM's is less sensitive to changes in the YTM.

- YTM = C → Par Value Bond
- YTM > C → Discount Bond
- YTM < C → Premium Bond

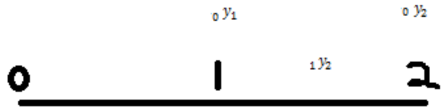
Realised Compound Yield (Y)

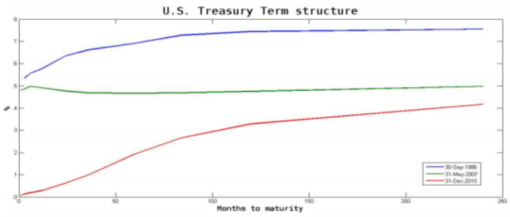
This measure is useful when the reinvestment rate ≠ YTM. We collect all cash flows at the maturity of the bond. *E.g. Bond FV = $100, T=2, C = 10%, P = $96.62, YTM = 12%, R = 10%. CF @ T = 2 = 110 + 11 = 121.* $Y = \sqrt[2]{\frac{121}{96.62}} - 1 = 11.9\%$

Week 2 – The Term Structure of Interest Rates

Term Structure of Interest Rates

If we were to <u>obtain a fixed interest rate</u> for an investment today <u>until time 't'</u>, this is called the't' spot rate – denoted as y_t
Together, <u>these spot rates make</u> up the <u>term structure of interest rates</u> or the pure yield curve. It's generally upward sloping due to liquidity risk.





We can think of interest rates as *(sort of)* prices of future CF's, like other prices it's determined by S/D. We use these different maturity spot rates to set up our arbitrage trades from week 1 & are the appropriate rates to discount future CF's. $P_0 = \frac{c}{1+y_1} + \frac{c}{(1+y_2)^2} + \frac{FV+c}{(1+y_3)^3}$. All competing bonds have identical HPR's

Inferring the Term Structure

Market bond prices imply the term structure.

Zero Coupon Bonds

$$P_0 = \frac{FV}{(1+y_t)^t} \rightarrow \rightarrow \rightarrow y_t = \left(\frac{FV}{P_0}\right)^{\frac{1}{t}}$$

Coupon Bonds

$$P_0 = \frac{c}{1+y_1} + \frac{FV+c}{(1+y_2)^2}$$

We have 1 equation with 2 unknowns. We must back out the spot rates in an iterative manner called bootstrapping.

1. Find y_1 from a 1 year zero coupon bond
2. Substitute this into the equation to find y_2

A general equation: $y_2 = \sqrt{\frac{FV+c}{P_2 - \frac{c}{1+y_1}}} - 1$

Example – Bond A, B, C

***Bond A** = 90.91, FV = 100, T = 1. **Bond B** = -79.72, FV =100, C = 0, T = 2. **Bond C** = 95.78, FV = 100, C = 10%, T = 2*

$$Bond\ A = 90.91 = \frac{100}{1+y_1} \rightarrow y_1 = \frac{100}{90.91} - 1 = 10\%$$

$$Bond\ B = 79.72 = \frac{100}{(1+y_2)^2} \rightarrow y_2 = \sqrt{\frac{100}{79.72}} - 1 = 12\%$$

Using the 2 spot rates we've bootstrapped, we calculate the arbitrage free price of Bond C *(=96.78 > 95.78).*

Arbitrage Strategy: **Buy Underpriced Bond C & Short Sell a Combination of Bond A & B**

We need short sell the overpriced synthetic bond & ensure that we have -\$10 *(at t=1)* & -\$110 *(at t=2).*