Geometry

Geometry is the branch of mathematics that deals with shape size, position and properties of figures. The word came from the Greek words “ge”, which means “earth”, and “metres”, “measurement”.

Geometrical shapes can be two-dimensional (having only length and width), and also three-dimensional (length x width x height).

Below are described the properties and ways of measurement of several geometrical shapes.

Bi-dimensional

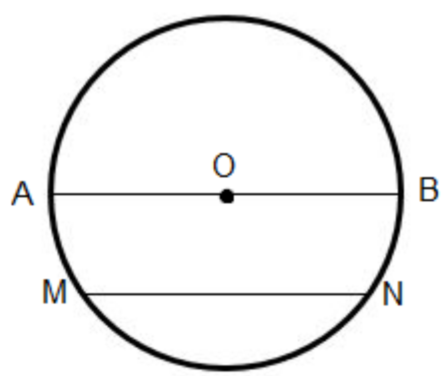
The Circle

A circle comprises all the points located at a given distance from a fixed point, which we call "the centre of the circle". For example, if we choose a point O and set a distance r (say r = 2 cm), then we locate O and start measuring 2 cm all around it.

If we draw ALL the points located at 2 cm from O, we obtain a circle with a radius of 2 cm. The radius (pl. radii) is the distance from the centre of the circle to any point on the circle.

Two radii that form an 180^o angle (a straight line) unites two opposite points on the circle. This line is called a diameter, and it always crosses the centre of the circle.

For example, in the circle below, segment AO is a radius, and AB is a diameter, but MN is not a diameter because it does not pass through the centre.



radius: AO = OB = 2 cm
diameter: AB = AO + OB = 4 cm

The length of a circle is called circumference.

$c = 2 \times \pi \times r$

Here, r is the radius, and π is approximated as 3,14.

In the case of the circle above, whose radius is 2 cm, the circumference will be:

$c = 2 \times \pi \times 2$

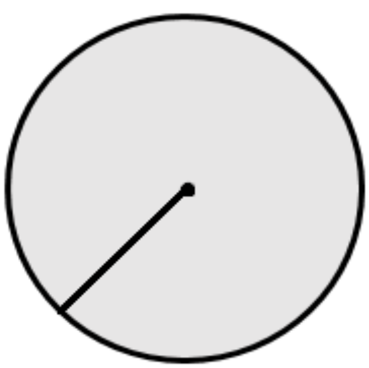
$c = 4 \times \pi$

$c = 4 \times 3.14$

$c = 12.56$ cm

If we consider all the points inside the circle, those points form a disc. The circle only has length, while the disc also has an area. For example, a hula hoop is a circle, but a Frisbee is a disc. However, in layman's terms, there is no clear distinction between a circle and a disc, and people use the term "circle" when talking about area, too.

The area of a circle is



r = 2 cm

$A = \pi \times r^2$

For example, let us calculate the area of the circle above, whose radius is 2 cm:

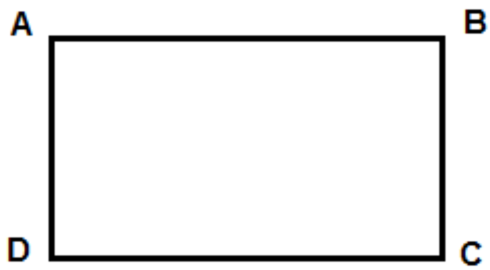
$A = \pi \times 2^2$

$A = 3.14 \times 4$

$A = 12.56\ cm^2$

The Rectangle

A rectangle has four sides, where the opposites are parallel and equal (or congruent, in



technical terms) and four right angles.

In the rectangle above, A, B, C and D are the corners (the four right angles).

AB = CD = the length of the rectangle

AD = BC = the width of the rectangle

The perimeter of a rectangle is easy to calculate, by adding together the lengths of the four sides, or, more simply, by adding the length and the width together, and doubling the sum.

$P = 2 \times (l + w)$

In the case of the rectangle above,

$P = 2 \times (AD + AB)$

Example: A football field is 30 m long and 20 m wide. Calculate the perimeter of the field.

The field is actually a rectangle, whose length and width are given in the data above. So we calculate the perimeter of the field using the formula for the perimeter of a rectangle:

$P = 2 \times (l \times w)$

$P = 2 \times (30 + 20)$

$P = 2 \times 50$

$P = 100$ m

If the question said that the players have to circle the field running 5 times, and you were required to calculate what distance the players would run, you should simply multiply the perimeter by 5.

d (distance) = 100 x 5

d = 500 m

In order to calculate the area of the rectangle, multiply the width by the length.

A = w x l

For the rectangle above,

A = AB x AD

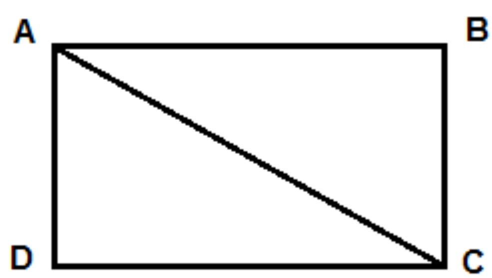
Example:

Let us calculate the area of the football field in the previous question:

A = 20 x 30

A = 600 m^2

Another thing that can be useful or necessary to calculate in a rectangle is the diagonal line. The diagonal divides the rectangle in two right-angled triangles. Therefore, we can determine the



diagonal by using Pythagoras's formula:

Example:

In the rectangle ABCD, AB = 12 cm and BC = 5 cm. What is the distance from point A to point C?

$AC^2 = AB^2 + BC^2$

$AC^2 = 12^2 + 5^2$

$AC^2 = 144 + 25$

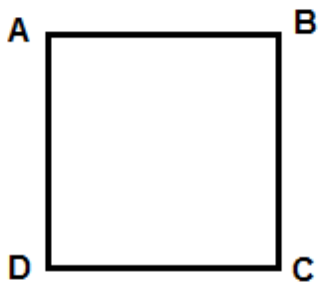
$AC^2 = 169$

$AC = \sqrt{169}$

AC = 13

The Square

Squares are just like rectangles, but all their sides are equal. So, in order to calculate the



perimeter, just multiply a side by 4. Let us consider s = side.

AB = BC = CD = DA = s (side)

P = 4 x s

Example:

Danny has to cut a paper square with a side of 8 cm. What is the total length of the cut?

Actually, what Danny cuts is the perimeter of the square.

So the cut will be side x 4.

P = 4 x s

P = 4 x 8

P = 32 cm

The area of a square is the square root (hence the name of “square root”) of the side.

$A = s^2$

Example:

A square window has a side of 50 cm. Calculate the area of the window.

The window is actually a square, so we use the formula for the area of a square:

$A = s^2$

$A = 50^2$

$A = 2500\ cm^2$

Also, if you need to determine the diagonal line of a square, you can use Pythagoras's theorem, or remember a simple formula.

$d = s \times \sqrt{2}$

Example:

A chess board is 30 cm long. Calculate the distance from one corner of the board to the opposite corner (the diagonal line).

$d = s \times \sqrt{2}$

$d = 30\sqrt{2}$

If we cannot remember the formula, we can just use Pythagoras:

$d^2 = 30^2 + 30^2$

$d^2 = 900 + 900$

$d^2 = 1800$

$d = \sqrt{1800}$

$d = \sqrt{(10^2 \times 3^2 \times 2)}$

The squared numbers can get in front of the radical, but they lose their power:

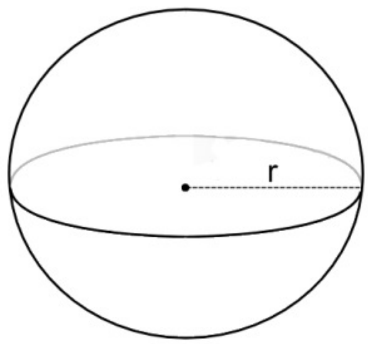
$d = 10 \times 3 \times \sqrt{2}$

$d = 30\sqrt{2}$

Tri-dimensional

The sphere

A sphere is composed of all the points "in space" located at an equal distance from a point called "the centre of the sphere".



Examples of spheres: a bauble, a volleyball etc.

The distance from the centre to any point on the sphere is called radius (r). The diameter, just like in the case of circles, is composed of two radii at 180^o (in other words, a diameter is the straight line between two points on the sphere that goes through the centre).

d = 2 x r

The area of the sphere can be calculated using the formula

$A = 4 \times \pi \times r^2$

Example:

The area of a sphere whose radius is 2 cm would be:

$A = 4 \times \pi \times 2^2$

A = 4 x 3.14 x 4

A = 8 x 3.14

$A = 25.12\ cm^2$

The space inside the sphere (defined as all the points located at smaller distances from the centre of the sphere than the radius) represents the volume of the sphere.

We can use the following formula to calculate the volume:

$V = 4 \times \pi \times r^3 / 3$

Example:

If r = 2, then the volume is:

$V = 4 \times \pi \times 2^3 / 3$

$V = 4 \times 3.14 \times 8 / 3$

(In calculations, we can safely eliminate 3.14 and 3, and estimate the result as slightly over 32 cm^3.)

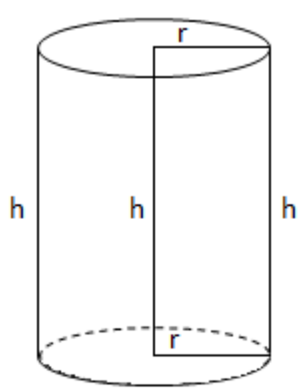
$V = 32 \times 3.14 / 3$

$V = 100.48 / 3$

$V = 33.49\ cm^3$

The cylinder

If we put several circles of the same radius one on top of the other, with the centres in collinear position, we obtain a cylinder. The number of circles we "put on top of one another" gives the height of the cylinder.



The cylinder has two identical circular bases. Therefore, we can infer that the volume of the cylinder will be given by the area of the base, multiplied by the height.

The area of the base is the area of a circle, i.e., $= \pi \times r^2$.

$V = \pi \times r^2 \times h$

Example:

Let us calculate the volume of a cylinder whose radius is 3 m (r = 3) and whose height is 5 m (h = 5).

$V = \pi \times 3^2 \times 5$

$V = \pi \times 9 \times 5$

$V = 3.14 \times 45$

$V = 141.3\ m^3$

Also, the area of the cylinder is the area of the two bases, plus the lateral area.

Since the base is a circle, we know that its area is

$A = \pi \times r2$

A cylinder has two bases, so the area becomes

$A = 2 \times \pi \times r2$

The lateral area is the length of the base multiplied by the height, because the lateral side of the cylinder is basically a rectangle. Think of the cylinder as a tin can, and the lateral side as a label that covers the can entirely, less the lid and the base. If you take the label down, you will see that it has the shape of a rectangle. So its area is length x width.

The length of the so-called rectangle is actually the length of the circle at the base. As discussed above, the length of the circle (the circumference) is $2 \times \pi \times r$, which means that the lateral area will be

$A = 2 \times \pi \times r \times h$

Therefore, the total area of the cylinder is

$A = 2 \times \pi \times r^2 + 2 \times \pi \times r \times h$

This can be factored out and reduced to

$A = 2 \times \pi \times r \times (r + h)$

In conclusion, even if you can't remember the formulas for the area and volume of the cylinder, you can still infer them using the formulas for the length and area of the circle.

Example:

Calculate the area of a cylinder, if r = 3 m and h = 5 m.

$A = 2 \times \pi \times 3 \times (3 + 5)$

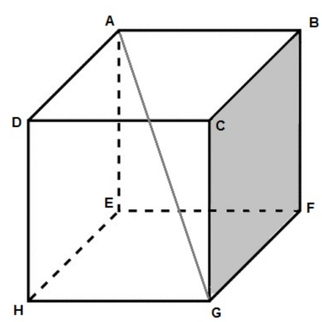
$A = 2 \times 3.14 \times 3 \times 8$

$A = 3.14 \times 48$

$A = 150.72\ m^2$

The Cube

A cube is a tri-dimensional body whose faces are squares. A cube has four faces, eight angles, and 12 edges. All the edges of a cube are equal, and all faces or sides are congruent (the faces are identical squares).



For example, in the cube above, A, B, C, E, F, G, H are angles or corners of the cube (in technical language, they are called vertices, sg. vertex).

AB = BC = CD = AD = EF = FG = GH = EH = DH = AE = BF = CG = edges

ABCD, EFGH, DCGH, BCGF, ABFE and ADHE are faces or sides of the cube.

AG is the diagonal. Other diagonals, that have not been drawn in this cube, could be DF, BH or CE.

If you need to calculate the perimeter of a face, simply use the formula for perimeter from the square; here, e stands for edge.

$P = 4 \times e$

The same goes for area. The area of a face is the area of a square, so

$a = e^2$

(I used a small "a" for area because it is the area of a single face; a capital "A" will be used for the area of the whole cube)

Since the cube has six faces, then its total area will be the area of a face multiplied by 6.

$A = e^2 \times 6$

Example:

A carver has to calculate the area of a cubic wood sculpture whose length, width and height are 0.5 meters each, in order to determine how much paint he needs to cover it.

Calculate the area of the sculpture.

In order to calculate the area, we first calculate the area of a "wall". It is better to transform the meters into centimetres, to avoid working with decimals.

0.5 m = 50 cm

$a = 50^2$

$a = 2500\ cm^2$

The cube has four "walls", so

$A = 2500 \times 6$

$A = 15000\ cm^2$

$15000\ cm^2 = 1.5\ m^2$

The volume of a cube can be calculated by multiplying the area of a face with an adjacent edge. Since all edges are equal in cubes, then the volume will be:

$V = (e^2) \times e$

$V = e^3$

Example:

A cubic tank is 12 meters high. How much water can be stored in the tank?

In order to calculate how much water (or anything else) can be stored inside the cube, we need to calculate the volume of the cube (the space inside it). We know that the edge is 12 m.

$V = e^3$

$V = 12^3$

$V = 1728\ m^3$

Sometimes, it might be necessary to calculate the diagonal line of a cube (going from one corner to the opposite one). This can be done using the formula:

$d = e\sqrt{3}$

So, in the case of our cube, $AG = AE\sqrt{3}$

The diagonal can also be determined using Pythagoras's theorem in the triangle AEG.

$AG^2 = AE^2 + EG^2$

AE is an edge, which we generally know the value or length of.

EG is the diagonal of a face, and since the face is a square, the diagonal is edge$\sqrt{2}$.

Example:

The edge of a cube is 3 cm. Calculate the diagonal of the cube.

Using the formula, we can quickly say that

$d = e\sqrt{3}$

$d = 3\sqrt{3}$

Otherwise, if we use Pythagoras's theorem,

$d^2 = s^2 + (s\sqrt{2})^2$

$d^2 = 3^2 + (3\sqrt{2})^2$

$d^2 = 9 + 9 \times 2$

$d^2 = 18$

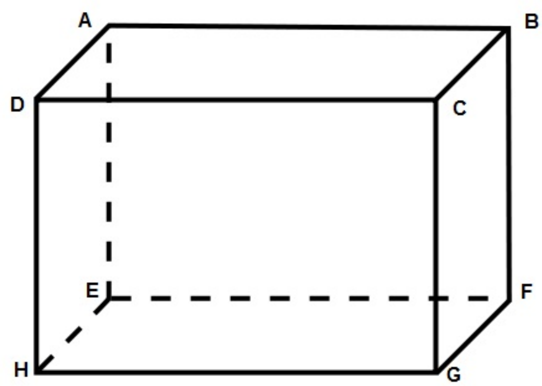
$d = \sqrt{18}$

$d = 3\sqrt{3}$

The Cuboid

A cuboid looks like a cube, with the difference that its faces are rectangles, not squares.

The area of a cuboid is the total area of its faces. Each two opposing faces are congruent, so they have the same area. Therefore, if we want to determine the overall area of the cuboid, we need the area of one face of each identical pair.



In the cuboid above,

ABCD = EFGH

AEHD = BFGC

ABFE = DCGH

The area of the cuboid is

$A = 2 \times A_{ABCD} + 2 \times A_{AEHD} + 2 \times A_{ABFE}$

$A = 2 \times (A_{ABCD} + A_{AEHD} + A_{ABFE})$

Example:

A cardboard box has the following dimensions: length = 60 cm, width = 20 cm, height = 30 cm. Calculate the area of the box.

The box is actually a cuboid. Let us calculate the area of each side in an identical pair:

$A_1 = 60 \times 20$

$A_1 = 1200\ cm^2$

$A_2 = 20 \times 30$

$A_2 = 600\ cm^2$

$A_3 = 60 \times 30$

$A_3 = 1800\ cm^2$

The total area will be:

$A = 2 \times (A_1 + A_2 + A_3)$

$A = 2 \times (1200 + 600 + 1800)$

$A = 2 \times 3600$

$A = 7200\ cm^2$

To calculate the volume of a cuboid, we simply multiply the three dimensions of the cuboid, the length, width and height.

In the case of the cuboid above,

V =AB x BC x CG = CD x AE x FG etc.

Example:

The floor of a room is 6 by 4 meters. Also, the room is 2.5 meters tall. Calculate the volume of the room.

$V = 6 \times 4 \times 2.5$

$V = 6 \times 10$

$V = 60\ m^3$