Performance Analysis of OFDM Systems with Phase Noise

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Abstract - *In this paper, we investigate the phase noise properties and the impacts caused by phase noise on an OFDM system, we derive the approximate expressions for BER performance in AWGN channel with various modulations techniques. The performance degradation is analyzed by graphs of BER against the number of sub-carriers. Thus the choice of the number of sub-carriers involves a tradeoff between mitigation of multi-path delay spread and reducing phase noise effects. Simulation results also show that phase noise effects performance for BPSK, QPSK & 16PSK constellation which lead to the conclusion that a phase noise mitigation mechanism is needed to obtain acceptable performance.*

Key Words: **OFDM, Phase Noise, ICI, BER, CPE, AWGN.**

1. Introduction

Orthogonal frequency division multiplexing (OFDM) is a multicarrier transmission technique, which divides the available spectrum into many carriers, each one being modulated by a low rate data stream. Practical implementations of OFDM technology use an inverse fast Fourier transform (IFFT) to generate a digitized version of the composite time domain signal. OFDM has been widely adopted and implemented in wire and wireless communication, such as digital subscriber line (DSL), digital terrestrial TV broadcasting (DVB-T), IEEE 802.11a wireless local area networks (WLANs), European high performance local area network (HIPERLAN/2) and possibly future 4G networks [1]–[4]. The disadvantage of OFDM, however, is its sensitive to both carrier frequencies offset and phase noise [5]–[6].

Frequency offset is a deterministic phenomenon, which is usually caused by the different carrier frequencies of the transmitter and the receiver, or by Doppler shift. On the other hand, phase noise is a random process caused by frequency fluctuation at the receiver and transmitter oscillators. Phase noise causes leakage of DFT which subsequently destroys the orthogonalities among subcarrier signals, leading to significant performance degradation. The principles of analyzing the frequency offset effects have been described in many papers [5], [7], hence in our contribution, only the performance analysis of OFDM systems with phase noise problem will be discussed.

Phase noise causes significant degradation of the performance of OFDM systems. The effects of phase noise on OFDM systems have been intensively investigated in the literature [5], [8]–[9]. Phase noise is normally modeled as an ideal phase modulation in the oscillator's signal. Phase noise in an OFDM system results from the instabilities in both oscillators of the transmitter and receiver. Without loss of generality, in this study the local oscillator in the receiver will be considered as the phase noise source.

Because of the random nature of phase noise, its effect is harder to be analyzed. In the literature, we will focus on the Wiener process assumption. The effect of Wiener phase noise has been expressed as a loss in the ratio between the signal power and the statistical average of the ICI [5]–[6], [10], wherein AWGN channel was considered in [10], while [5]–[6] included fading channel. However, it is both interesting and useful to find the precise relation between the bit error rate (BER) or symbol error rate (SER) and the phase noise. The original work done by Tomba [11], which is based on [12], considers only the BER for the AWGN channel and ignores the comparison with simulation results.

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In this paper, we propose a new method to derive the expressions for BER performance in AWGN channel with the presence of phase noise. Simulation results validate all expressions we have derived and show clearly the necessity of phase noise mitigation mechanism.

The rest of the paper is organized as follows. We describe the system model in section 2, where phase noise and OFDM system models are presented. In section 3, different effects of phase noise are discussed. In Section 4, numerical simulation results are presented. Finally we draw some conclusions in section 5.

2. System Model

A. Phase Noise Model

Phase noise $\phi(t)$, generated at both transmitter and receiver oscillators, can be described as a continuous Brownian motion process given by

$$\phi(t) = \int_0^t u(t)dt \qquad (1)$$

with zero mean and variance $2\pi\beta t$, where β denotes the phase noise linewidth, i.e., frequency spacing between 3dB points of its Lorentzian power spectral density function [5], [12]. Such noise has independent Gaussian increments [12], which, from a spectral point of view, can be presented as a Wiener process [8], [13]. To better characterize phase noise, Demir et al. [14] developed a unifying theory using a nonlinear method that provides more accurate description. With such a method [14, Remark 7.1], phase noise $\phi(t)$ is shown to converge, asymptotically with time, to a Gaussian random process having a constant mean, a variance increasing linearly with time, and a correlation function that satisfies

$$E\{[\phi(t) - \phi(t+\tau)]^2\} = 2\pi\beta|\tau| \qquad (2)$$

Furthermore, as indicated in [15], the aforementioned properties suggests a discrete Markov process which, in an OFDM system, can describe the phase noise on the *n*th sample of the *m*th symbol as

$$\phi_m(n) = \phi_{m-1}(N-1) + \sum_{i=-N_g}^{n} u[m(N+N_g)+i] = \sum_{i=0}^{m(N+N_g)+N_g+n} u(i) \qquad (3)$$

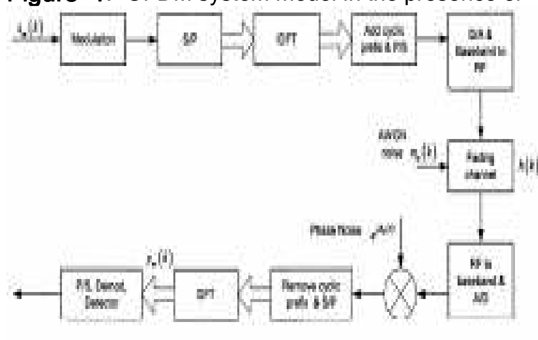
where u(i)'s denote mutually independent Gaussian random variables with zero mean and variance $\sigma_u^2 = 2\pi\beta T / N$, and T denotes the OFDM symbol length. In particular, (3) reduces to

$\phi_0(n) = \sum_{i=-N_g}^{n} u(i)$ for m = 0($\phi_m(n)$=0, when m<0).

B. OFDM System Model

Since the effects of phase noise on OFDM systems are particular interest, perfect frequency and timing synchronizations are assumed. Thus, in the presence of phase noise, the OFDM system model is shown in figure 1.

Figure 1. OFDM system model in the presence of



phase noise

The OFDM system model over a slowly time varying multipath fading channel with phase noise can be described as

$$y_m(n) = e^{j\phi(n)} \sum_{l=0}^{L-1} h_l x_m(n-l) + n(n) \qquad 0 \le n \le N-1 \qquad (4)$$

where h_l's $0 \le l \le L-1$ are complex-valued Rayleigh fading random variables, and n(n)'s (0 ≤ n ≤ N-1) are independent complex-valued Gaussian random variables with zero mean and variance for both real and imaginary components, while are the time domain transmitted symbols. L is the length of the time-domain channel impulse response (CIR). After discarding the cyclic prefix (CP) and performing an FFT at the receiver, we obtain the received data frame in the frequency domain:

$$Y_m(k) = X_m(k)H_m(k)C_m(0) + ICI_m(k) + N(k) \qquad (5)$$

with

$$C_m(k) = \frac{1}{N}\sum_{n=0}^{N-1} e^{j\left[\frac{2\pi nk}{N} + \phi_m(n)\right]} \qquad (6)$$

$ICI_m(k)$ is defined as

$\sum_{i=0, j\neq k}^{N-1} X_m(l)H_m(l)C_m(k-l)$, and N(k) is the DFT transform of n(n). Since DFT is linear and does not change the noise energy, N(k) is still a Gaussian random variable with zero mean and variance σ_n^2. Note that common phase error (CPE) and ICI are represented by $C_m(0)$ and $ICI_m(k)$ respectively. If

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phase noise does not exist, Eqations (5) and (6) reduce to

$$Y_m(k) = X_m(k)H_m(k) + N(k) \text{ and } C_m(k) = \delta(k)$$

respectively.

3. Effects of Phase Noise

The effect of phase noise leads to significant loss of OFDM systems. Phase noise destroys orthogonalities among OFDM subcarrier signals. There are two major problems affected by phase noise: phase shift and ICI. For the purpose of convenience, we will drop the subscript *m*, i.e., our discussion is restricted in one OFDM symbol. The effects of phase noise on OFDM signals are illustrated in figure 2, where phase noise corrupts the 16-QAM constellations with both CPE and ICI. It's shown that phase noise forces the desired signal into a wrong decision area, and worsens the BER performance accordingly.

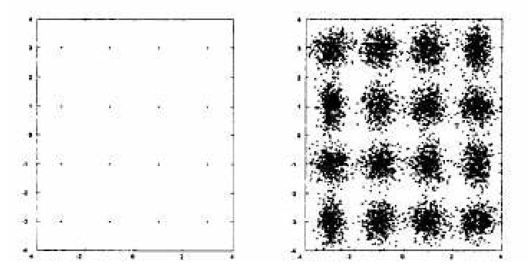

Figure 2. Phase noise effects on16-QAM modulated OFDM signals (a) original 16-QAM constellations (b) 16-QAM constellations corrupted by phase noise

A. The Effect on Phase shift

Common Phase Error (CPE), indicated by $C(0)$, causes the rotation of the desired signals, is invariant within one OFDM symbol, although it is varying from symbol to symbol. From Equation (6), we have,

$$C(0) = \frac{1}{N}\sum_{n=0}^{N-1} e^{j\phi(n)} = Ae^{j\theta} \qquad (7)$$

where A denotes the amplitude of CPE and θ is a random variable denoting phase shift, which is the major effect of CPE when phase noise is small. The probability density function (pdf) of θ which will be needed for analyzing its effect on BER performance, as

$$f_\theta(x) \approx \frac{1}{\sqrt{2\pi\sigma_\theta}} e^{-(x^2/2\sigma_\theta^2)} \qquad (8)$$

where $\sigma_\theta^2 = \frac{7}{12}N\sigma_u^2 = \frac{7}{6}\pi\beta T.$

B. The Effect on ICI

ICI plays an important part in BER analysis. It is usually treated in AWGN channel as a Gaussian distributed random variable [11]–[12], whereas in fading channel, such approximation is not appropriate in some scenario.

4. Simulation Results

To demonstrate the performance analysis of OFDM systems with phase noise, computer simulations were performed. A very powerful and useful engineering software package is Matlab by MathWorks [16]-[17]. It has many useful digital signal processing functions and features, which will prove to be useful in an OFDM simulation. An OFDM system was modeled using Matlab to allow various parameters of the system to be varied and tested.

A. BER Performance in AWGN Channel

In this section, We study the performance analysis in the presence of phase noise for OFDM in AWGN channel, i.e., $H(k) = 1$ for all k. When we transmit the signal through an AWGN channel, receive it and check the errors. The simulation is based on multiple signal-to-noise-ratio (SNR); meaning that the signal is received for various SNR values and error check is performed. The following figure shows BER VS. SNR graph under the AWGN channel conditions the results from the simulations. The simulations were performed using 200 carriers and generated using a 512-point IFFT.

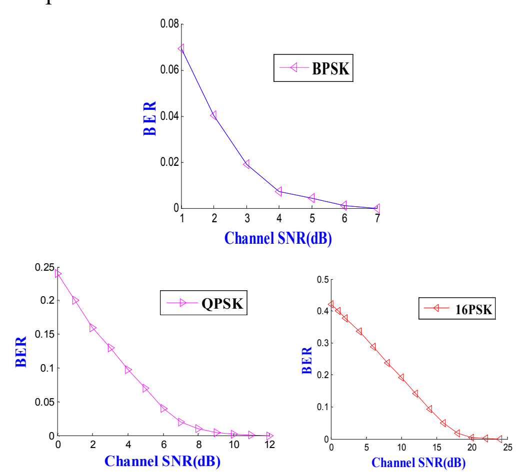

Figure 3. The BER vs. SNR for OFDM using BPSK, QPSK, 16PSK respectively

The results show that using QPSK the transmission can tolerate a SNR of >11-13 dB. The bit error rate BER gets rapidly worse as the SNR drops below 6 dB. However, using BPSK allows the BER to be improved in a noisy channel, at the expense of

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transmission data capacity. Using BPSK the OFDM transmission can tolerate a SNR of >7-8 dB. In a low noise link, using 16PSK can increase the capacity. If the SNR is >24 dB 16PSK can be used, doubling the data capacity compared with QPSK.

B. BER Performance vs Number of Sub-Carriers

The BER performance is much dependent on the choice of number of sub-carrier On the other hand, phase noise effects are worse as the number of sub-carriers is increased. Thus the need for a tradeoff was realized and was examined using the simulation runs. BER vs. number of sub-carriers curves have been provided. Figure 4 shows the simulation result for the effect of the number of sub-carriers on the BER performance due to phase noise. This shows the concrete relationship between the number of sub-carriers and phase noise effects on system performance.

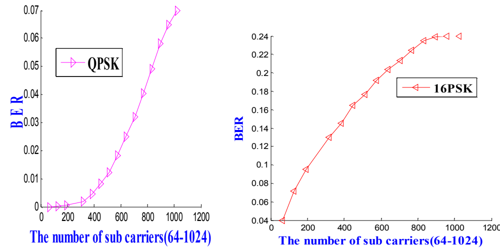

Figure 4. The number of sub-carriers vs. BER, when SNR=10dB

C. Phase Noise Effects Performance for BPSK, QPSK & 16PSK Constellation

The Additive White Gaussian Noise (AWGN) and the presence of phase noise corrupted the transmitted signal and this resulted in a different received constellation than the original constellation. For small SNR values the calculated error rate was quite large and ISI was produced due the relative high power of noise. The following figure shows the simulation results for phase noise effects. The received BPSK, QPSK & 16PSK constellation is shown in Figure 5, Figure 6 and Figure 7.

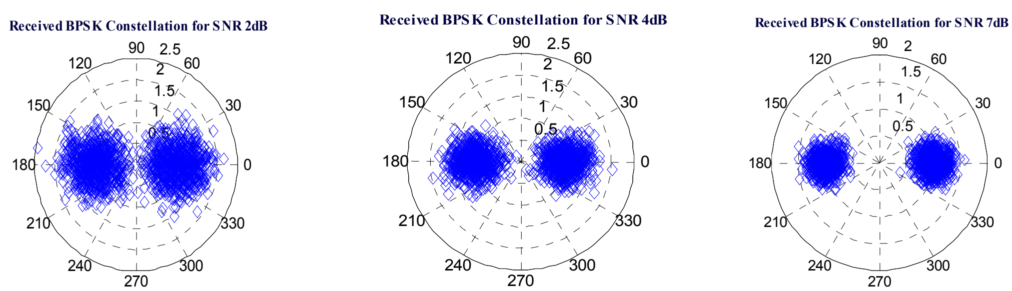

Figure 5. Received BPSK constellation for corresponding SNR values of 2dB, 4dB and 7dB

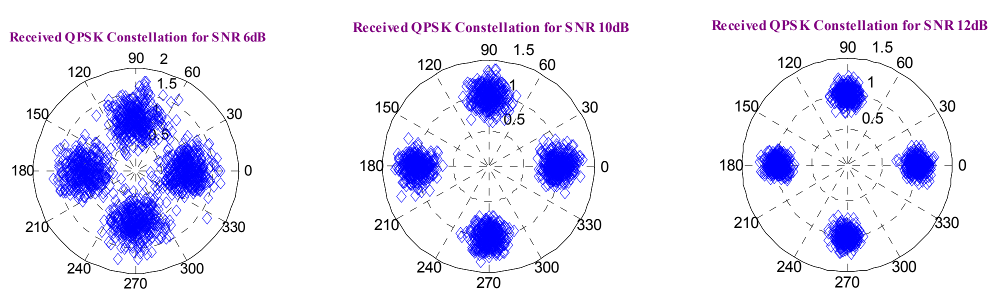

Figure 6. Received QPSK constellation for corresponding SNR values of 6dB, 10dB and 12dB

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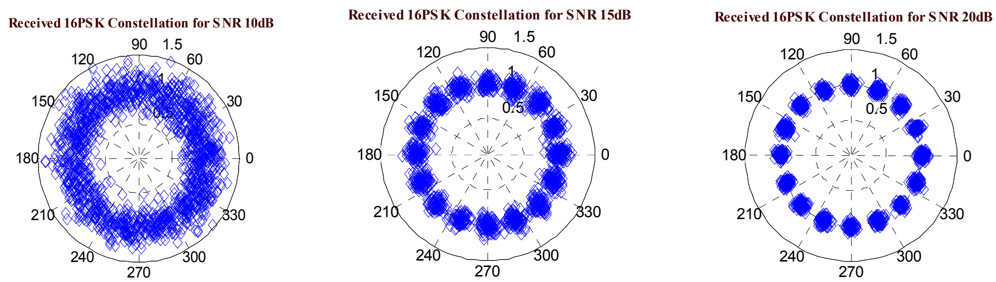

Figure 7. Received 16PSK constellation for corresponding SNR values of 10dB, 15dB and 20dB

It is clear that as the SNR is increased the received constellation gets less affected by the noise; hence there will be fewer errors. The error, which is a symbol error, is calculated by comparing the original constellation with the received constellation. However, for low values of SNR we have ISI introduced by the noise at the receiver side. As SNR was increased the error rate was decreasing, as expected. In fact, for a SNR value greater than 8dB for BPSK, the error was zero.

5. Conclusions

Phase noise causes significant degradation of the performance of OFDM systems. The effects of phase noise on the performance of OFDM systems have been analytically evaluated. The expressions for CPE and ICI caused by phase noise have been derived. It is shown that both CPE and ICI depend on the overall spectrum of each weighted group of subcarriers rather than on the spectrum of each individual subcarrier. Simulation results show that the methodology presented in this work is accurate and can be used as references in practice.

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