Robust H_∞ Filter Design of Uncertain Descriptor Systems with Discrete and Distributed Delays

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***Abstract*—The robust H_∞ filtering problem for a class of continuous-time uncertain linear descriptor systems with time-varying discrete and distributed delays is investigated. The time delays are assumed to be constant and known. The uncertainties under consideration are norm-bounded, and possible time-varying, uncertainties. Sufficient condition for the existence of an H_∞ filter is expressed in terms of strict linear matrix inequalities (LMIs). Instead of using decomposition technique, a unified form of LMIs is proposed to show the exponential stability of the augmented systems. The condition for assuring the stability of the "fast" subsystem is implied from the unified form of LMIs, which is shown to be less conservative than the characteristic equation based conditions or matrix norm-based conditions. The suitable filter is derived through a convex optimization problem. A numerical example is given to show the effectiveness of the method.**

***Index Terms*—Descriptor systems, discrete delay, distributed delay, linear matrix inequality (LMI), robust H_∞ filter, stability.**

I. Introduction

SIGNAL estimation has received significant attention in the past decades [1], [18]. Current efforts on this topic can be divided into two classes: the Kalman filtering approach and the H_∞ filtering approach.

In the Kalman filtering approach, the systems disturbances are assumed to be Gaussian noises with known statistics; see, for example, for linear systems [23], [26], and [29] and for linear descriptor systems [4], [6], and [7]. When the systems noise sources are assumed to be arbitrary signals with bounded energy (or average power), the H_∞ filtering approach provides a guaranteed noise attention level. One of its main advantages is the fact that it is insensitive to the exact knowledge of the statistics of the noise signals. Several methods are proposed to solve the H_∞ filtering problem [2], [20], [32].

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When there exist parameter uncertainties in the systems model, robust H_∞ filtering can provide a powerful signal estimation. It designs an asymptotically stable filter, based on an uncertain signal model, which ensures that the filtering error dynamics is asymptotically stable and that the L_2-induced gain from the noise signals to the filtering error remains bounded by a prescribed level for all allowed uncertainties. Many results regarding robust H_∞ filtering are obtained; see, e.g., [16], [23], and [26].

Time-delays are frequently encountered in practical systems such as engineering and biological systems [13]. Their existence may induce instability, oscillation, and poor performance [34]. Time delays also arise in several signal processing such as multipath propagation [14], telemanipulation systems [25], data communication in high-speed internet [27], and network control systems [15]. When one designs an H_∞ filter, the time-delay must be taken into account in order to make the system work in the expected performance. Otherwise, the system may collapse in the presence of time delays. Recently, there have been increasing interests in designing an H_∞ filter for time-delay systems. For example, in [24], an H_∞ filter design for precisely known systems with a single time-delayed measurement was proposed. In [28], based on an algebraic Riccati matrix inequality approach, the robust H_∞ filtering was investigated for uncertain linear systems with delayed states and outputs. In [8], robust H_∞ filtering for uncertain linear systems with multiple time-varying state delays was considered, and a delay-independent sufficient condition was given in the form of linear matrix inequalities (LMIs). In [10], based on a descriptor model transformation, a delay-dependent H_∞ filtering design was proposed for linear systems with constant time delay. The filter obtained was of the Luenberger observer type. The results in [10] were extended to a system with time-varying delay and improved by employing the Parks [22] inequality for the bounding of cross terms [12].

As is well known, one can use the time-delay model to describe the so-called "lossless propagation phenomena" [13]. These models can be further transformed to descriptor systems with time delay; see, e.g., [21]. The descriptor systems with time delay are systems of a more general type. It is of significance to consider the H_∞ filtering problem for these kinds of systems. To the best of authors' knowledge, the problem was only investigated in [11], where the approach was based on the decomposition technique, and the filter was of the Luenberger observer type. In [11], the uncertainties under consideration were polytopic ones. It is difficult to extend the results in [11] to other types of uncertainties such as norm-bounded ones. The distributed delay was not considered in [11].

This paper will be concerned with the robust H_∞ filtering for a class of uncertain linear descriptor systems with discrete and distributed delays. The uncertainties are norm-bounded ones. The sufficient condition for the existence of an H_∞ filter will be expressed in terms of strict LMIs. Instead of using the decomposition technique, a unified form of LMIs will be proposed to show the exponential stability of the augmented systems. The condition for assuring the exponential stability of the "fast" subsystem will be implied from the unified form of LMIs, which is shown to be less conservative than the characteristic equation-based conditions or matrix norm-based conditions. The suitable filter will be derived through a convex optimization problem. A numerical example will be finally given to show the effectiveness of the method.

Notation: R^n denotes the n-dimensional Euclidean space, $R^{n\times m}$ is the set of $n\times m$ real matrices, I is the identity matrix of appropriate dimensions, and $\|\cdot\|$ stands for either the Euclidean vector norm or its induced matrix 2-norm. The notation $X>0$ (respectively, $X\geq 0$) for $X\in R^{n\times n}$ means that the matrix X is a real symmetric positive definite (respectively, positive semi-definite). C_0 denotes the set of all continuous functions from $[-\tau',0]$ to R^n. $\lambda_{\max}(X)$ ($\lambda_{\min}(X)$) denotes the maximum (minimum) eigenvalue of the real symmetric matrix X. $\mathrm{tr}(Y)$ denotes the trace of a matrix Y. $\mathrm{Re}(s)$ denotes the real part of a complex numbers. "$*$" denotes the entries implied by symmetry of a matrix. For a vector function $g(t)\in L_2[0,\infty)$, its norm is defined as

$$\|g(t)\|_2=\sqrt{\int_0^\infty \|g(t)\|^2\,dt}.$$

II. Preliminaries

Consider the following uncertain descriptor system with discrete and distributed delays:

$$\begin{aligned}E\dot{x}(t)&=(A+\Delta A(t))\,x(t)+(A_\tau+\Delta A_\tau(t))\,x(t-\tau)\\&\quad+\int_{t-h}^{t}(A_h+\Delta A_h(s))\,x(s)ds+B_1w(t) &(1)\\ y(t)&=(C+\Delta C(t))\,x(t)+(C_\tau+\Delta C_\tau(t))\,x(t-\tau)\\&\quad+\int_{t-h}^{t}(C_h+\Delta C_h(s))\,x(s)ds+B_2w(t) &(2)\\ z(t)&=Lx(t) &(3)\\ x(t)&=\varphi(t),\qquad t\in[-\tau',0],\tau'=\max\{\tau,h\} &(4)\end{aligned}$$

where $x(t)\in R^n$ is the system state, $w(t)\in R^q$ is the external disturbance signal that belongs to $L_2[0,\infty)$, $y(t)\in R^r$ is the measurement, and $z(t)\in R^p$ is the signal to be estimated. $\tau>0$ and $h>0$ are constants describing the magnitude of delay time. $\varphi(t)\in C_0$ denotes the initial function. E, A, A_τ, A_h, B_1, C, C_τ, C_h, B_2, and L are known constant matrices of appropriate dimensions. $\Delta A(t)$, $\Delta A_\tau(t)$, $\Delta A_h(t)$, $\Delta C(t)$, $\Delta C_\tau(t)$, and $\Delta C_h(t)$ denote the parameter uncertainties that satisfy

$$\begin{bmatrix}\Delta A(t) & \Delta A_\tau(t) & \Delta A_h(t)\\ \Delta C(t) & \Delta C_\tau(t) & \Delta C_h(t)\end{bmatrix}=\begin{bmatrix}D_a\\ D_b\end{bmatrix}F(t)[E_a\ \ E_b\ \ E_c] \qquad (5)$$

where D_a, D_b, E_a, E_b, and E_c are known matrices of appropriate dimensions, and $F(t)$ is an unknown, piecewise continuous time-varying matrix that satisfies $\|F(t)\|\leq 1$. Throughout this paper, we assume that $\mathrm{rank}(E)=q\leq n$.

Similar to [31], we introduce a definition on regularity and nonimpulsiveness of the system (1).

Definition 1: The descriptor system (1) [with $w(t)=0$] is said to be regular and impulse free if $(E, A+\Delta A(t))$ is regular and impulse free.

Consider a linear filter with full order as

$$\begin{aligned}E\frac{d\bar{x}(t)}{dt}&=A_f\bar{x}(t)+K_fy(t) &(6)\\ \bar{z}(t)&=L\bar{x}(t) &(7)\\ \bar{x}(0)&=0 &(8)\end{aligned}$$

where $\bar{x}$ is the state estimate, and the constant matrices A_f and K_f are filter parameters to be determined.

To begin with the study of the state estimation problem, we define the state error variable as

$$e(t)=x(t)-\bar{x}(t). \qquad (9)$$

Then, from (1), (2), and (6), $e(t)$ satisfies the following dynamics:

$$\begin{aligned}E\dot{e}(t)&=A_fe(t)\\&\quad+[A-A_f+\Delta A(t)-K_f(C+\Delta C(t))]\,x(t)\\&\quad+[A_\tau-K_fC_\tau+\Delta A_\tau(t)-K_f\Delta C_\tau(t)]\,x(t-\tau)\\&\quad+\int_{t-h}^{t}[A_h-K_fC_h+\Delta A_h(s)-K_f\Delta C_h(s)]\\&\qquad\times x(s)ds+(B_1-K_fB_2)w(t). \qquad (10)\end{aligned}$$

From (1), (3), and (10), we have the following augmented system:

$$\begin{aligned}E_f\dot{x}_f(t)&=(A_{af}+\Delta A_{af}(t))\,x_f(t)\\&\quad+(A_{\tau f}+\Delta A_{\tau f}(t))\,x_f(t-\tau)\\&\quad+\int_{t-h}^{t}(A_{hf}+\Delta A_{hf}(s))\,x_f(s)ds\\&\quad+B_fw(t) &(11)\\ z_f(t)&=L_fx_f(t) &(12)\\ x_f(t)&=\begin{bmatrix}\varphi(t)\\ \varphi(t)\end{bmatrix},\quad t\in[-\tau',0] &(13)\end{aligned}$$

where $z_f(t)$ is the estimation error, and

$$\begin{aligned}x_f&=\begin{bmatrix}x(t)\\ e(t)\end{bmatrix}\\ E_f&=\begin{bmatrix}E & 0\\ 0 & E\end{bmatrix}\\ A_{af}&=\begin{bmatrix}A & 0\\ A-A_f-K_fC & A_f\end{bmatrix}\\ A_{\tau f}&=\begin{bmatrix}A_\tau & 0\\ A_\tau-K_fC_\tau & 0\end{bmatrix}\\ A_{hf}&=\begin{bmatrix}A_h & 0\\ A_h-K_fC_h & 0\end{bmatrix}\end{aligned}$$

$$\begin{aligned} B_f &= \begin{bmatrix} B_1 \\ B_1 - K_f B_2 \end{bmatrix} \\ L_f &= [0 \quad L] \\ \Delta A_{af}(t) &= \begin{bmatrix} \Delta A(t) & 0 \\ \Delta A(t) - K_f \Delta C(t) & 0 \end{bmatrix} \\ \Delta A_{\tau f}(t) &= \begin{bmatrix} \Delta A_\tau(t) & 0 \\ \Delta A_\tau(t) - K_f \Delta C_\tau(t) & 0 \end{bmatrix} \\ \Delta A_{hf}(t) &= \begin{bmatrix} \Delta A_h(t) & 0 \\ \Delta A_h(t) - K_f \Delta C_h(t) & 0 \end{bmatrix}. \end{aligned} \tag{14}$$

From (5), $\Delta A_{af}(t)$, $\Delta A_{\tau f}(t)$, and $\Delta A_{hf}(t)$ can be expressed as

$$[\Delta A_{af}(t) \;\; \Delta A_{\tau f}(t) \;\; \Delta A_{hf}(t)] = D_f F(t) [E_{af} \quad E_{bf} \quad E_{cf}] \tag{15}$$

where

$$\begin{aligned} D_f &= \begin{bmatrix} D_a \\ D_a - K_f D_b \end{bmatrix} \\ E_{af} &= [E_a \quad 0] \\ E_{bf} &= [E_b \quad 0] \\ E_{cf} &= [E_c \quad 0]. \end{aligned} \tag{16}$$

The filtering design problem to be addressed is stated as follows.

Robust H_∞ Filtering Problem: For a given $\gamma > 0$, design a full-order linear filter of the form (6)–(8) such that the augmented system (11)–(13) is regular, impulse-free, and internally exponentially stable, namely, there exist $\alpha > 0$ and $\beta > 0$ such that the solution x_f of (11) and (13) with $w(t) = 0$ satisfies $\|x_f(t)\| \leq \alpha \sup_{-\tau' \leq s \leq 0} \|\varphi(s)\| e^{-\beta t}$ under zero initial condition, and for any nonzero $w(t) \in L_2[0, \infty)$, $z_f(t)$ satisfies $\|z_f(t)\|_2 \leq \gamma \|w(t)\|_2$.

III. Robust H_∞ Performance Analysis

In this section, we will concentrate our attention on the robust performance analysis for system (11)–(13). The following lemmas are useful in the proof of Theorem 1.

Lemma 1: Suppose that piecewise continuous real square matrices $A(t)$, X, and $Q > 0$ satisfy

$$A^T(t) X + X^T A(t) + Q < 0 \tag{17}$$

for all t. Then, the following hold.

1) $A(t)$ and X are invertible.
2) $\|A^{-1}(t)\| \leq \delta$ for some $\delta > 0$.

Lemma 2: Suppose that a positive continuous function $f(t)$ satisfies

$$f(t) \leq \zeta_1 \sup_{t-\tau \leq s \leq t} f(s) + \zeta_2 e^{-\varepsilon t} \tag{18}$$

where $\varepsilon > 0$, $\zeta_1 < 1$, $\zeta_2 > 0$, and $\tau > 0$. Then, $f(t)$ satisfies

$$f(t) \leq \sup_{-\tau \leq s \leq 0} f(s) e^{-\xi_0 t} + \frac{\zeta_2 e^{-\xi_0 t}}{1 - \zeta_1 e^{\xi_0 \tau}}, \qquad t \geq 0 \tag{19}$$

where $\xi_0 = \min\{\varepsilon, \xi\}$, and $0 < \xi < -(1/\tau) \ln \zeta_1$.

The proofs of Lemmas 1 and 2 are given in the Appendix.

Based on Lemmas 1 and 2, we are now in a position to state and establish the following theorem that gives sufficient conditions assuring a guaranteed γ level of noise attenuation to the filtering error systems of (11)–(13).

Theorem 1: Given scalars $0 < a < 1$ and $\gamma > 0$. Suppose that matrices P, $Q > 0$, and $T > 0$ are such that

$$P E_f = E_f^T P^T \geq 0 \tag{20}$$

and we also have (21), shown at the bottom of the page, where $s \in [t - h, t]$. Then, the augmented system (11)–(13) is regular, impulse-free, and internally exponentially stable and satisfies a prescribed H_∞ norm upper bound constraint, that is, $\|z_f(t)\|_2 \leq \gamma \|w(t)\|_2$ for any nonzero $w(t) \in L_2[0, \infty)$.

The proof of Theorem 1 can be found in the Appendix.

For the cases when $A_{hf} = \Delta A_{hf} \equiv 0$ and $A_{\tau f} = \Delta A_{\tau f} \equiv 0$, by Theorem 1, the following corollaries are easily obtained, respectively.

Corollary 1: Consider system (11)–(13), where $A_{hf} = \Delta A_{hf} \equiv 0$. For a given scalar $\gamma > 0$, if there exist matrices P, $Q > 0$, and $T > 0$ such that

$$P E_f = E_f^T P^T \geq 0 \tag{22}$$

we also have (23), shown at the bottom of the page. Then, the augmented system (11)–(13) is regular, impulse-free, and internally exponentially stable and satisfies a prescribed H_∞ norm upper bound constraint, that is, $\|z_f(t)\|_2 \leq \gamma \|w(t)\|_2$ for any nonzero $w(t) \in L_2[0, \infty)$.

$$\begin{bmatrix} \begin{matrix} (A_{af} + \Delta A_{af}(t))^T P^T + P(A_{af} + \Delta A_{af}(t)) \\ + L_f^T L_f + Q + T \end{matrix} & P(A_{\tau f} + \Delta A_{\tau f}(t)) & hP(A_{hf} + \Delta A_{hf}(s)) & PB_f \\ (A_{\tau f} + \Delta A_{\tau f}(t))^T P^T & -aQ & 0 & 0 \\ h(A_{hf} + \Delta A_{hf}(s))^T P^T & 0 & -(1-a)Q & 0 \\ B_f^T P^T & 0 & 0 & -\gamma^2 I \end{bmatrix} < 0 \tag{21}$$

$$\begin{bmatrix} (A_{af} + \Delta A_{af}(t))^T P^T + P(A_{af} + \Delta A_{af}(t)) + L_f^T L_f + Q + T & P(A_{\tau f} + \Delta A_{\tau f}(t)) & PB_f \\ (A_{\tau f} + \Delta A_{\tau f}(t))^T P^T & -Q & 0 \\ B_f^T P^T & 0 & -\gamma^2 I \end{bmatrix} < 0 \tag{23}$$

Corollary 2: Consider system (11)–(13), where $A_{\tau f} = \Delta A_{\tau f} \equiv 0$. For a given scalar $\gamma > 0$, if there exist matrices P, $Q > 0$, and $T > 0$ such that

$$PE_f = E_f^T P^T \geq 0 \tag{24}$$

we then have (25), shown at the bottom of the page. Then, the augmented system (11)–(13) is regular, impulse-free, and internally exponentially stable and satisfies a prescribed H_∞ norm upper bound constraint, that is, $\|z_f(t)\|_2 \leq \gamma \|w(t)\|_2$ for any nonzero $w(t) \in L_2[0, \infty)$.

Remark 1: It is worth pointing out that for the time-invariant parameter uncertainty case, T in Corollary 1 can be set as a zero matrix. Therefore, in the case of $E = I$ and where the parameter uncertainties are time invariant, Corollary 1 is an LMI form of [28, Lemma 4]. Moreover, if one only considers the stability of nominal systems, [31, Th. 1] is easily covered by Corollary 1. Therefore, Theorem 1 can be viewed as an extension of the existing results to the descriptor systems with time-varying uncertainties and discrete and distributed delays. However, our analysis procedure is different from that in [31], and the derived stability in our paper is exponential stability.

Remark 2: From the proof of Theorem 1, it can be found that LMI-based condition (71) is a sufficient condition for guaranteeing stability of the "fast" subsystem (75). In the existing literature [11], [17], to show stability of the "fast" subsystem, the following norm upper bound based condition was extensively used

$$\left\|\tilde{A}_{22}^{-1}(t)\tilde{A}_{\tau 22}(t)\right\| + \int_{t-h}^{t} \left\|\tilde{A}_{22}^{-1}(t)A_{h22}(s)\right\| ds \leq 1 - \delta < 1 \tag{26}$$

where $\delta > 0$ is a sufficiently small real number. Since (21) implies (71), no decomposition of the system matrices is needed to apply our method. However, to determine (26), it is necessary to decompose the system matrices first, which may lead to the complexity and fallibility of the method. In addition, the following simple example shows that (71) may also lead to much less conservative results than that by using (26). Consider a simple (75) with parameter matrices

$$\tilde{A}_{22}(t) = I, \quad \tilde{A}_{\tau 22}(t) = \begin{bmatrix} -0.5 & 1 \\ 0 & 0.3 \end{bmatrix}$$
$$\tilde{A}_{h22}(s) = \begin{bmatrix} 0.2 & 0 \\ 1 & 0.5 \end{bmatrix}, \quad h = 0.5.$$

Obviously, no conclusion on the stability of the "fast" system can be made by (26), whereas it is guaranteed by (71) through choosing $a = 0.5$.

Remark 3: For a special descriptor system with distributed delay terms, which is an equivalent system of the state-space system (1) in [30], stability analysis was given based on a generalized Lyapunov functional. From the view point of descriptor system theory, it can be seen from [30, proof of Th. 2.1] that only the stability of the state of the slow subsystem was studied, although it is enough for the paper [30]. For a general class of descriptor systems with delays, the stability of the two subsystems, namely, the "slow" subsystem and the "fast" subsystem, must be addressed in order to show the stability of the whole system. Instead of using decomposition technique, based on both a generalized Lyapunov functional (54) and an algebraic function (78), a unified form of LMIs was proposed in our paper to show the exponential stability of the augmented system (11)–(13). The condition for assuring the stability of the "fast" subsystem was implied from the unified form of LMIs, which has been shown in Remark 2 to be less conservative than the characteristic equation-based conditions or matrix norm-based conditions. It should be noted that for [30, (5)] in the case of $A_d(s) = 0$ and $C_1 = 0$ or the case of $d = h$, Corollary 2 in our paper has the equivalent condition as the one in [30, (8) in Th. 2.1], whereas Corollary 2 can determine not only the stability of $x(t)$ but also the stability of $y(t)$ directly from the information of parameter matrices of [30, (5)].

IV. Robust H_∞ Filter Design

After finishing some necessary preparations in the last section, we can now devote ourselves to the design of filter parameters A_f and K_f. The expected filter parameters will be expressed in terms of the solutions of a set of LMIs, which can be realized in the following theorem.

Theorem 2: Given scalars $0 < a < 1$ and $\gamma > 0$. If there exist matrices P_o, P_1, P_2, P_3, Y, and $R > 0$ and scalars $\varepsilon_i > 0$ $(i = 1, 2)$, $\sigma > 0$ such that

$$P_1 E = E^T P_1^T \geq 0 \tag{27}$$
$$P_2 E = E^T P_2^T \geq 0 \tag{28}$$
$$P_3 E = 0 \tag{29}$$

we then have (30), shown at the bottom of the next page. Then, the robust H_∞ filtering problem for system (1)–(4) is solvable. Moreover, the parameters of the designed filter are given by

$$A_f = P_2^{-1} P_o, \quad K_f = P_2^{-1} Y. \tag{31}$$

In order to prove Theorem 2, the following lemma is needed.

Lemma 3 [3]:

1) For any real vectors x, y, and a real matrix $P > 0$ of appropriate dimensions

$$2x^T y \leq x^T P^{-1} x + y^T P y.$$

$$\begin{bmatrix} (A_{af} + \Delta A_{af}(t))^T P^T + P(A_{af} + \Delta A_{af}(t)) + L_f^T L_f + Q + T & hP(A_{hf} + \Delta A_{hf}(s)) & PB_f \\ h(A_{hf} + \Delta A_{hf}(s))^T P^T & -Q & 0 \\ B_f^T P^T & 0 & -\gamma^2 I \end{bmatrix} < 0 \tag{25}$$

2) Let A, D, E, and $F(t)$ be real matrices of appropriate dimensions with $\|F(t)\| \le 1$. Then, for any scalar $\varepsilon > 0$, the following inequality holds:

$$DF(t)E + E^T F^T(t) D^T \le \varepsilon^{-1} DD^T + \varepsilon E^T E.$$

Proof of Theorem 2: Define (32), shown at the bottom of the page.

By (15), Π can be expressed as the last equation shown at the bottom of the page. Using Lemma 3, we obtain (33), shown at the bottom of the next page, where $\varepsilon_i > 0\ (i = 1, 2)$. By Schur complements, it can be shown that $\Pi' < 0$ is equivalent to (34), shown at the bottom of the next page. Let

$$P = \begin{bmatrix} P_1 & 0 \\ P_3 & P_2 \end{bmatrix}, \quad Q = \begin{bmatrix} R & 0 \\ 0 & \sigma I \end{bmatrix} \tag{35}$$

where $P_1, P_2, P_3 \in R^{n\times n}$, $R > 0$, and $\sigma > 0$ is a scalar. Obviously, (20) holds if and only if (27), (28), and (29) hold. Substituting the above P and Q into (34) yields (36), shown at the bottom of the page after the next page. Note that (36) implies that

$$A_f^T P_2^T + P_2 A_f + L^T L + \sigma I < 0.$$

By Lemma 1, it is easy to see that P_2 is invertible.

Define $P_o = P_2 A_f$ and $Y = P_2 K_f$. Since $\sigma > 0$, one can see that (36) is equivalent to (30). Therefore, if (30) holds, then $\Pi' < 0$. Noting that Π' is a block 4×4 matrix, it is obvious to get (37), shown at the bottom of the page after the next page. Defining $T = (1/2)\lambda_{\min}(-\Pi')I$, from (33), (37), and by Theorem 1, we can complete our proof. □

In light of Lemma 3 and Corollary 1 or Corollary 2, we reach the following conclusions.

$$\begin{bmatrix}
A^T P_1^T + P_1 A + \varepsilon_1 E_a^T E_a + R & A^T P_3^T + A^T P_2^T - P_o^T - C^T Y^T & P_1 A_\tau + \varepsilon_1 E_a^T E_b & hP_1 A_h & P_1 B_1 & P_1 D_a & P_1 D_a \\
* & P_o^T + P_o + L^T L + \sigma I & P_3 A_\tau + P_2 A_\tau - Y C_\tau & hP_3 A_h + hP_2 A_h - hY C_h & P_3 B_1 + P_2 B_1 - Y B_2 & P_3 D_a + P_2 D_a - Y D_b & P_3 D_a + P_2 D_a - Y D_b \\
* & * & -aR + \varepsilon_1 E_b^T E_b & 0 & 0 & 0 & 0 \\
* & * & * & -(1-a)R + \varepsilon_2 h^2 E_c^T E_c & 0 & 0 & 0 \\
* & * & * & * & -\gamma^2 I & 0 & 0 \\
* & * & * & * & * & -\varepsilon_1 I & 0 \\
* & * & * & * & * & * & -\varepsilon_2 I
\end{bmatrix} < 0 \tag{30}$$

$$\Pi = \begin{bmatrix}
\begin{matrix}(A_{af} + \Delta A_{af}(t))^T P^T + P(A_{af} + \Delta A_{af}(t)) \\ + L_f^T L_f + Q\end{matrix} & P(A_{\tau f} + \Delta A_{\tau f}(t)) & hP(A_{hf} + \Delta A_{hf}(s)) & PB_f \\
(A_{\tau f} + \Delta A_{\tau f}(t))^T P^T & -aQ & 0 & 0 \\
h(A_{hf} + \Delta A_{hf}(s))^T P^T & 0 & -(1-a)Q & 0 \\
B_f^T P^T & 0 & 0 & -\gamma^2 I
\end{bmatrix}. \tag{32}$$

$$\Pi = \begin{bmatrix}
A_{af}^T P^T + PA_{af} + L_f^T L_f + Q & PA_{\tau f} & hPA_{hf} & PB_f \\
A_{\tau f}^T P^T & -aQ & 0 & 0 \\
hA_{hf}^T P^T & 0 & -(1-a)Q & 0 \\
B_f^T P^T & 0 & 0 & -\gamma^2 I
\end{bmatrix}
+ \begin{bmatrix} PD_f \\ 0 \\ 0 \\ 0 \end{bmatrix} F(t) [E_{af} \quad E_{bf} \quad 0 \quad 0] + \begin{bmatrix} E_{af}^T \\ E_{bf}^T \\ 0 \\ 0 \end{bmatrix} F^T(t) \left[D_f^T P^T \quad 0 \quad 0 \quad 0\right]$$
$$+ \begin{bmatrix} PD_f \\ 0 \\ 0 \\ 0 \end{bmatrix} F(s) [0 \quad 0 \quad hE_{cf} \quad 0] + \begin{bmatrix} hE_{cf}^T \\ 0 \\ 0 \end{bmatrix} F^T(s) \left[D_f^T P^T \quad 0 \quad 0 \quad 0\right].$$

Corollary 3: Consider system (1)–(4) without the distributed delay term. For a given scalar $\gamma > 0$, if there exist matrices P_o, P_1, P_2, P_3, Y, and $R > 0$ and scalars $\varepsilon > 0$, $\sigma > 0$ such that

$$P_1 E = E^T P_1^T \geq 0 \quad (38)$$
$$P_2 E = E^T P_2^T \geq 0 \quad (39)$$
$$P_3 E = 0 \quad (40)$$

we get (41), shown at the bottom of the next page. Then, the robust H_∞ filtering problem for system (1)–(4) is solvable. Moreover, the parameters of the designed filter are given by

$$A_f = P_2^{-1} P_o, \quad K_f = P_2^{-1} Y. \quad (42)$$

Corollary 4: Consider system (1)–(4) without the discrete delay term. For a given scalar $\gamma > 0$, if there exist matrices P_o, P_1, P_2, P_3, Y, and $R > 0$ and scalars $\varepsilon_i > 0$ $(i = 1, 2)$, $\sigma > 0$ such that

$$P_1 E = E^T P_1^T \geq 0 \quad (43)$$
$$P_2 E = E^T P_2^T \geq 0 \quad (44)$$
$$P_3 E = 0 \quad (45)$$

we get (46), shown at the bottom of the page after the next page. Then, the robust H_∞ filtering problem for system (1)–(4) is solvable. Moreover, the parameters of the designed filter are given by

$$A_f = P_2^{-1} P_o, \quad K_f = P_2^{-1} Y. \quad (47)$$

Remark 4: In Theorem 2 and the resulting corollaries, equality constraints are included, which will lead to numerical problems when checking such nonstrict LMI conditions since equality constraints are often fragile and usually not met perfectly [31]. For the case that $\text{rank}(E) = q < n$, there exists a matrix $\Phi \in R^{n \times (n-q)}$ with $\text{rank}(\Phi) = n - q$ such that $\Phi^T E = 0$. Define $P_i = E^T \Theta_i + Z_i \Phi^T$ $(i = 1, 2)$ and $P_3 = Z_3 \Phi^T$, where $\Theta_i \in R^{n \times n}$ $(i = 1, 2)$ is positive definite, and $Z_i \in R^{n \times (n-q)}$ $(i = 1, 2, 3)$. Obviously, $P_i E = E^T P_i^T \geq 0$ $(i = 1, 2)$ and $P_3 E = 0$ hold. Denote (30) as the inequality for Theorem 2 after substituting $P_i = E^T \Theta_i + Z_i \Phi^T$ $(i = 1, 2)$ and $P_3 = Z_3 \Phi^T$ into (30)'. Then, the solution of P_i $(i = 1, 2, 3)$ satisfying (27), (28), and (30) can be transformed into the solution of Θ_i $(i = 1, 2)$ and Z_i $(i = 1, 2, 3)$ satisfying the strict LMI (30)'. From (31), we can finally obtain the parameters of the designed filter as $A_f = (E^T \Theta_2 + Z_2 \Phi^T)^{-1} P_o$ and $K_f = (E^T \Theta_2 + Z_2 \Phi^T)^{-1} Y$. For Corollaries 1 and 2, we can use the same procedure to transform the corresponding nonstrict LMIs into strict LMIs.

Remark 5: If a and γ are fixed, the upper bound of h that guarantees the solution of the problem (30)' is feasible can be

$$\Pi \leq \begin{bmatrix} A_{af}^T P^T + P A_{af} + L_f^T L_f + Q & P A_{\tau f} & h P A_{hf} & P B_f \\ A_{\tau f}^T P^T & -aQ & 0 & 0 \\ h A_{hf}^T P^T & 0 & -(1-a)Q & 0 \\ B_f^T P^T & 0 & 0 & -\gamma^2 I \end{bmatrix}$$
$$+ \varepsilon_1^{-1} \begin{bmatrix} P D_f \\ 0 \\ 0 \\ 0 \end{bmatrix} \begin{bmatrix} D_f^T P^T & 0 & 0 & 0 \end{bmatrix} + \varepsilon_1 \begin{bmatrix} E_{af}^T \\ E_{bf}^T \\ 0 \\ 0 \end{bmatrix} \begin{bmatrix} E_{af} & E_{bf} & 0 & 0 \end{bmatrix}$$
$$+ \varepsilon_2^{-1} \begin{bmatrix} P D_f \\ 0 \\ 0 \\ 0 \end{bmatrix} \begin{bmatrix} D_f^T P^T & 0 & 0 & 0 \end{bmatrix} + \varepsilon_2 \begin{bmatrix} 0 \\ 0 \\ h E_{cf}^T \\ 0 \end{bmatrix} \begin{bmatrix} 0 & 0 & h E_{cf} & 0 \end{bmatrix} \triangleq \Pi' \quad (33)$$

$$\begin{bmatrix} A_{af}^T P^T + P A_{af} + \varepsilon_1 E_{af}^T E_{af} + L_f^T L_f + Q & P A_{\tau f} + \varepsilon_1 E_{af}^T E_{bf} & h P A_{hf} & P B_f & P D_f & P D_f \\ * & -aQ + \varepsilon_1 E_{bf}^T E_{bf} & 0 & 0 & 0 & 0 \\ * & * & -(1-a)Q + \varepsilon_2 h^2 E_{cf}^T E_{cf} & 0 & 0 & 0 \\ * & * & 0 & -\gamma^2 I & 0 & 0 \\ * & * & * & * & -\varepsilon_1 I & 0 \\ * & * & * & * & * & -\varepsilon_2 I \end{bmatrix} < 0. \quad (34)$$

solved. Generally, a can be chosen as 0.5. In addition, when a and h are fixed, the smallest γ describing the disturbance attenuation level can be solved from the following optimization problem:

$$\begin{aligned}&\text{Minimize } \vartheta\\ &\text{Subject to}: \Theta_i > 0,\ R > 0\\ &\qquad \varepsilon_i > 0\ (i=1,2) \ \text{ and (30)'}\end{aligned} \tag{48}$$

and $\gamma = \sqrt{\vartheta^*}$, ϑ^* is the optimal value of problem (48). Furthermore, it is shown by the following example that appropriately adjusting the parameter a may lead to less conservative results.

$$\begin{bmatrix} A^TP_1^T + P_1A + \varepsilon_1 E_a^TE_a + R & A^TP_3^T + A^TP_2^T - A_f^TP_2^T - C^TK_f^TP_2^T & P_1A_\tau + \varepsilon_1 E_a^TE_b & 0 & hP_1A_h & 0 & P_1B_1 & P_1D_a & P_1D_a \\ * & A_f^TP_2^T + P_2A_f + L^TL + \sigma I & P_3A_\tau + P_2A_\tau - P_2K_fC_\tau & 0 & hP_3A_h + hP_2A_h - hP_2K_fC_h & 0 & P_3B_1 + P_2B_1 - P_2K_fB_2 & P_3D_a + P_2D_a - P_2K_fD_b & P_3D_a + P_2D_a - P_2K_fD_b \\ * & * & -aR + \varepsilon_1 E_b^TE_b & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & * & -a\sigma I & 0 & 0 & 0 & 0 & 0 \\ * & * & * & * & -(1-a)R + \varepsilon_2 h^2 E_c^TE_c & 0 & 0 & 0 & 0 \\ * & * & * & * & * & -(1-a)\sigma I & 0 & 0 & 0 \\ * & * & * & * & * & * & -\gamma^2 I & 0 & 0 \\ * & * & * & * & * & * & * & -\varepsilon_1 I & 0 \\ * & * & * & * & * & * & * & * & -\varepsilon_2 I \end{bmatrix} < 0. \tag{36}$$

$$\begin{aligned}\Pi' &< \operatorname{diag}\left(-\lambda_{\min}(-\Pi')I \quad -\lambda_{\min}(-\Pi')I \quad -\lambda_{\min}(-\Pi')I \quad -\lambda_{\min}(-\Pi')I\right)\\ &< \operatorname{diag}\left(-\lambda_{\min}(-\Pi')I \quad 0 \quad 0 \quad 0\right).\end{aligned} \tag{37}$$

$$\begin{bmatrix} A^TP_1^T + P_1A + \varepsilon E_a^TE_a + R & A^TP_3^T + A^TP_2^T - P_o^T - C^TY^T & P_1A_\tau + \varepsilon_1 E_a^TE_b & P_1B_1 & P_1D_a \\ * & P_o^T + P_o + L^TL + \sigma I & P_3A_\tau + P_2A_\tau - YC_\tau & P_3B_1 + P_2B_1 - YB_2 & P_3D_a + P_2D_a - YD_b \\ * & * & -R + \varepsilon E_b^TE_b & 0 & 0 \\ * & * & * & -\gamma^2 I & 0 \\ * & * & * & * & -\varepsilon I \end{bmatrix} < 0. \tag{41}$$

V. EXAMPLE

Consider system (1)–(4) with parameters

$$E = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$
$$A = \begin{bmatrix} -2 & 0 & 0.5 \\ 0.1 & -0.9 & 0.2 \\ 0 & 0.5 & 0.3 \end{bmatrix}$$
$$A_\tau = \begin{bmatrix} 0.2 & 0.1 & 0 \\ 0.2 & 0 & 0.15 \\ 0.1 & -0.23 & 0.1 \end{bmatrix}$$
$$A_h = \begin{bmatrix} 0.1 & 0 & 0.1 \\ 0.03 & 0.1 & 0 \\ 0.1 & 0.02 & 0.2 \end{bmatrix}$$
$$B_1 = \begin{bmatrix} 0.2 & 0 & 0 \\ 0 & 0.2 & 0 \\ 0 & 0 & 0.2 \end{bmatrix}$$
$$C = [1 \quad 0 \quad 0]$$
$$C_\tau = [0 \quad 0.5 \quad 0.4], \quad C_h = [0.5 \quad 0.3 \quad 0.6]$$
$$B_2 = [0.3 \quad 0.3 \quad 0.3], \quad L = [1 \quad 0.7 \quad 0.8]$$
$$D_a = 0.1I, \quad E_a = E_b = E_c = I$$
$$D_b = [0.1 \quad 0.1 \quad 0.1], \quad h = 2.$$

Choose $\Phi = [0\ 0\ 1]^T$ and $a = 0.5$. For $\gamma = 1$, applying Theorem 2 and Remark 5, we can solve Θ_i $(i = 1, 2)$, Z_i $(i = 1, 2, 3)$, Y, and P_o as

$$\Theta_1 = \begin{bmatrix} 4.9370 & 0.7074 & 0.0000 \\ 0.7074 & -7.4751 & -6.8018 \\ 0.0000 & 0.0000 & 6.9976 \end{bmatrix}$$
$$\Theta_2 = \begin{bmatrix} 3.6741 & 0.3137 & 0.0000 \\ 0.3137 & 1.5250 & 0.0000 \\ 0.0000 & 0.0000 & 6.9976 \end{bmatrix}$$
$$Z_1 = \begin{bmatrix} -1.4050 \\ -2.0598 \\ -3.7196 \end{bmatrix}$$
$$Z_2 = \begin{bmatrix} 13.8112 \\ 6.4031 \\ -1.7201 \end{bmatrix}$$
$$Z_3 = \begin{bmatrix} -14.7180 \\ -7.1905 \\ 0.0270 \end{bmatrix}$$
$$Y = \begin{bmatrix} 0.0316 \\ 0.0884 \\ -0.6297 \end{bmatrix}$$
$$P_o = \begin{bmatrix} -5.0105 & -0.4229 & 0.6975 \\ -0.6393 & -1.0681 & -0.3930 \\ 0.0890 & -0.8358 & -1.7594 \end{bmatrix}.$$

Then, from (31), we can compute A_f and K_f as

$$A_f = \begin{bmatrix} -1.1727 & -1.7381 & -3.3248 \\ 0.0392 & -2.3829 & -3.8683 \\ -0.0517 & 0.4859 & 1.0229 \end{bmatrix}$$
$$K_f = [-1.2634 \quad -1.2191 \quad 0.3661]^T.$$

In fact, when a is chosen to be 0.5, one can find an upper bound of h that guarantees that the feasibility of problem (30)' is 2.72. However, if one chooses a as 0.4, the upper bound of h can be 2.76. By optimization algorithm (48), we can find that the smallest γ is 0.6397 for $h = 2$ and $a = 0.5$ and the corresponding A_f and K_f are

$$A_f = 10^3 \times \begin{bmatrix} -0.4428 & -0.4829 & 0.4388 \\ -6.8267 & -7.4751 & -6.8018 \\ -0.0016 & 0.0007 & 0.0027 \end{bmatrix}$$
$$K_f = [0.5731 \quad 6.7398 \quad 0.9808]^T.$$

VI. CONCLUSION

The robust H_∞ filtering problem has been addressed for continuous-time uncertain descriptor systems with discrete and distributed delays. The designed filter can guarantee that the filtering error system is regular, impulse-free, and exponentially stable and satisfies a prescribed H_∞ norm bound constraint. The decomposition-free method has been used to derive the LMI-based sufficient conditions, which can be efficiently solved by using an interior-point optimization algorithm.

$$\begin{bmatrix} A^TP_1^T + P_1A + \varepsilon_1 E_a^T E_a + R & A^TP_3^T + A^TP_2^T - P_o^T - C^TY^T & hP_1A_h & P_1B_1 & P_1D_a & P_1D_a \\ * & P_o^T + P_o + L^TL + \sigma I & hP_3A_h + hP_2A_h - hYC_h & P_3B_1 + P_2B_1 - YB_2 & P_3D_a + P_2D_a - YD_b & P_3D_a + P_2D_a - YD_b \\ * & * & -R + \varepsilon_2 h^2 E_c^T E_c & 0 & 0 & 0 \\ * & * & * & -\gamma^2 I & 0 & 0 \\ * & * & * & * & -\varepsilon_1 I & 0 \\ * & * & * & * & * & -\varepsilon_2 I \end{bmatrix} < 0 \quad (46)$$

APPENDIX

Proof of Lemma 1

Since $Q > 0$, there exists a scalar $\alpha > 0$ such that $Q \geq \alpha I$. Therefore, it follows from (17) that

$$A^T(t)X + X^T A(t) + \alpha I < 0. \quad (49)$$

Recalling the fact [9] that

$$\mathrm{Re}\lambda(N) \leq \frac{1}{2}\lambda_{\max}(N + N^T)$$

where N is a real square matrix, we obtain from (49) that

$$\mathrm{Re}\lambda\left(A^T(t)X\right) < -\frac{\alpha}{2}.$$

Hence, $A^T(t)X$ is invertible for all t. Consequently, $A^T(t)$ and X are invertible for all t. Similar to the proof of [33, Lemma 2.2], it is easy to prove that $\|A^{-1}(t)\| \leq \delta$ holds for some $\delta > 0$. Therefore, the proof is omitted. □

Proof of Lemma 2

From (18), we know that

$$f(t) \leq \zeta_1 \sup_{t-\tau \leq s \leq t} f(s) + \zeta_2 e^{-\xi_0 t}, \quad t \geq 0. \quad (50)$$

Next, we first prove that for any $\varepsilon_0 > 0$

$$f(t) < \sup_{-\tau \leq s \leq 0} f(s)e^{-\xi_0 t} + \frac{\zeta_2 e^{-\xi_0 t}}{1 - \zeta_1 e^{\xi_0 \tau}} + \varepsilon_0. \quad t \geq 0. \quad (51)$$

Note that

$$f(0) \leq \zeta_1 \sup_{-\tau \leq s \leq 0} f(s) + \zeta_2 < \sup_{-\tau \leq s \leq 0} f(s) + \frac{\zeta_2}{1 - \zeta_1 e^{\xi_0 \tau}} + \varepsilon_0.$$

If (51) is not true, then $\bar{t}$ exists such that

$$f(\bar{t}) = \sup_{-\tau \leq s \leq 0} f(s)e^{-\xi_0 \bar{t}} + \frac{\zeta_2 e^{-\xi_0 \bar{t}}}{1 - \zeta_1 e^{\xi_0 \tau}} + \varepsilon_0 \quad (52)$$

and

$$f(t) < \sup_{-\tau \leq s \leq 0} f(s)e^{-\xi_0 t} + \frac{\zeta_2 e^{-\xi_0 t}}{1 - \zeta_1 e^{\xi_0 \tau}} + \varepsilon_0, \quad t < \bar{t}. \quad (53)$$

In fact, for $t \in [-\tau, 0]$, we have

$$f(t) \leq \sup_{-\tau \leq s \leq 0} f(s) < \sup_{-\tau \leq s \leq 0} f(s)e^{-\xi_0 t} + \frac{\zeta_2 e^{-\xi_0 t}}{1 - \zeta_1 e^{\xi_0 \tau}} + \varepsilon_0.$$

Therefore, (53) holds for any $t \in [-\tau, \bar{t}]$. However, from (50), (52), and (53), we can see that

$$\begin{aligned} f(\bar{t}) &\leq \zeta_1 \sup_{\bar{t}-\tau \leq s \leq \bar{t}} f(s) + \zeta_2 e^{-\xi_0 \bar{t}} \\ &\leq \zeta_1 e^{\xi_0 \tau} \sup_{-\tau \leq s \leq 0} f(s)e^{-\xi_0 \bar{t}} \\ &\quad + \frac{\zeta_1 e_2^{\xi_0 \tau} \zeta_2 e^{-\xi_0 \bar{t}}}{1 - \zeta_1 e^{\xi_0 \tau}} + \zeta_1 \varepsilon_0 + \zeta_2 e^{-\xi_0 \bar{t}} \\ &< \sup_{-\tau \leq s \leq 0} f(s)e^{-\xi_0 \bar{t}} + \frac{\zeta_2 e^{-\xi_0 \bar{t}}}{1 - \zeta_1 e^{\xi_0 \tau}} + \varepsilon_0 \end{aligned}$$

which contradicts (52). By letting $\varepsilon_0 \to 0$ in (51), we obtain (19). □

Proof of Theorem 1

For $(t, x_{tf}) \in R \times C([-\tau', 0], R^n)$, where $x_{tf}(\theta) = x_f(t + \theta)$, $\theta \in [-\tau', 0]$, we define a generalized Lyapunov functional as

$$\begin{aligned} V(t, x_{tf}) &= x_f^T(t) P E_f x_f(t) + a \int_{t-\tau}^{t} x_f^T(s) Q x_f(s) ds \\ &\quad + \frac{1-a}{h} \int_{t-h}^{t} \int_{s}^{t} x_f^T(u) Q x_f(u) du ds. \end{aligned} \quad (54)$$

Taking the time derivative of $V(t, x_{tf})$ along the trajectory of system (11) yields

$$\begin{aligned} \dot{V}(t, x_{tf}) = &\, 2x_f^T(t) P \left(A_{af} + \Delta A_{af}(t)\right) x_f(t) \\ &+ 2x_f^T(t) P \left(A_{\tau f} + \Delta A_{\tau f}(t)\right) x_f(t-\tau) \\ &+ 2x_f^T(t) P \int_{t-h}^{t} \left(A_{hf} + \Delta A_{hf}(s)\right) x_f(s) ds \\ &+ 2x_f^T(t) P B_f w(t) + a x_f^T(t) Q x_f(t) \\ &- a x_f^T(t-\tau) Q x_f(t-\tau) + (1-a) x_f^T(t) Q x_f(t) \\ &- \frac{(1-a)}{h} \int_{t-h}^{t} x_f^T(s) Q x_f(s) ds + z_f^T(t) z_f(t) \\ &- \gamma^2 w^T(t) w(t) - z_f^T(t) z_f(t) + \gamma^2 w^T(t) w(t) \\ \leq &\, x_f^T(t) \Big[\left(A_{af} + \Delta A_{af}(t)\right)^T P^T \\ &\quad + P\left(A_{af} + \Delta A_{af}(t)\right) + L_f^T L_f + Q \Big] x_f(t) \\ &+ \int_{t-h}^{t} x_f^T(t) P \left(A_{hf} + \Delta A_{hf}(s)\right) \frac{h}{(1-a)} \\ &\quad \times Q^{-1} \left(A_{hf} + \Delta A_{hf}(s)\right)^T P^T x_f(t) ds \\ &+ 2x_f^T(t) P \left(A_{\tau f} + \Delta A_{\tau f}(t)\right) x_f(t-\tau) \\ &+ 2x_f^T(t) P B_f w(t) \\ &- a x_f^T(t-\tau) Q x_f(t-\tau) - \gamma^2 w^T(t) w(t) \\ &- z_f^T(t) z_f(t) + \gamma^2 w^T(t) w(t) \\ = &\, \frac{1}{h} \int_{t-h}^{t} \nu(t) U(t,s) \nu^T(t) ds \\ &- z_f^T(t) z_f(t) + \gamma^2 w^T(t) w(t) \end{aligned} \quad (55)$$

where

$$\begin{aligned} \nu(t) &= \begin{bmatrix} x_f^T(t) & x_f^T(t-\tau) & w^T(t) \end{bmatrix} \\ U(t,s) &= \begin{bmatrix} \Psi_0 & P\left(A_{\tau f} + \Delta A_{\tau f}(t)\right) & PB_f \\ * & -aQ & 0 \\ * & * & -\gamma^2 I \end{bmatrix} \\ \Psi_0 &= \left(A_{af} + \Delta A_{af}(t)\right)^T P^T + P\left(A_{af} + \Delta A_{af}(t)\right) \\ &\quad + P\left(A_{hf} + \Delta A_{hf}(s)\right) \frac{h^2}{(1-a)} Q^{-1} \\ &\quad \times \left(A_{hf} + \Delta A_{hf}(s)\right)^T P^T + L_f^T L_f + Q. \end{aligned} \quad (56)$$

By Schur complements, it is easy to see from (21) and (56) that

$$U(t,s) < 0.$$

Therefore, it follows from (55) that

$$\dot{V}(t,x_{tf}) \leq -z_f^T(t)z_f(t) + \gamma^2 w^T(t)w(t). \qquad (57)$$

Integrating both sides of above inequality from 0 to ∞ yields

$$\int_0^\infty z_f^T(t)z_f(t)dt \leq V(0,\varphi) - V(\infty,x_\infty) + \int_0^\infty \gamma^2 w^T(t)w(t)dt$$
$$\leq V(0,\varphi) + \int_0^\infty \gamma^2 w^T(t)w(t)dt \qquad (58)$$

which deduces, under zero initial condition, i.e., $\varphi(t)=0$, that

$$\int_0^\infty z_f^T(t)z_f(t)dt \leq \int_0^\infty \gamma^2 w^T(t)w(t)dt$$

that is, $\|z_f(t)\|_2 \leq \gamma\|w(t)\|_2$.

If the external disturbance $w(t)$ is zero, i.e., $w(t)\equiv 0$, then it follows from (55) that

$$\dot{V}(t,x_{tf}) \leq \frac{1}{h}\int_{t-h}^{t} \begin{bmatrix} x_f^T(t) & x_f^T(t-\tau)\end{bmatrix} U'(t,s) \begin{bmatrix} x_f(t) \\ x_f(t-\tau)\end{bmatrix} ds \qquad (59)$$

where

$$U'(t,s) = \begin{bmatrix} \Psi_1 & P\left(A_{\tau f}+\Delta A_{\tau f}(t)\right) \\ * & -aQ \end{bmatrix}$$
$$\Psi_1 = \left(A_{af}+\Delta A_{af}(t)\right)^T P^T + P\left(A_{af}+\Delta A_{af}(t)\right) + Q + P\left(A_{hf}+\Delta A_{hf}(s)\right)\frac{h^2}{(1-a)} \times Q^{-1}\left(A_{hf}+\Delta A_{hf}(s)\right)^T P^T.$$

By Schur complements and from (21), we can show that

$$U'(t,s) \leq -\mathrm{diag}(T \quad 0). \qquad (60)$$

Therefore, it follows from (59) and (60) that

$$\dot{V}(t,x_{tf}) \leq -\lambda\|x_f(t)\|^2 \qquad (61)$$

where $\lambda = \lambda_{\min}(T)$.

Define a new function as

$$W(t,x_{tf}) = e^{\varepsilon t}V(t,x_{tf}) \qquad (62)$$

and taking its time derivative yields

$$\dot{W}(t,x_{tf}) = \varepsilon e^{\varepsilon t}V(t,x_{tf}) + e^{\varepsilon t}\dot{V}(t,x_{tf}) \leq \varepsilon e^{\varepsilon t}V(t,x_{tf}) - \lambda e^{\varepsilon t}\|x_f(t)\|^2. \qquad (63)$$

Integrating both sides of (63) from 0 to t obtains

$$W(t,x_{tf}) - W(0,\varphi) \leq \int_0^t \varepsilon e^{\varepsilon s}V(s,x_{sf})ds - \lambda\int_0^t e^{\varepsilon s}\|x_f(s)\|^2 ds. \qquad (64)$$

By using the similar analysis method of [19], it can be seen from (54), (62), and (64) that, if ε is chosen small enough, a constant $\beta > 0$ can be found such that

$$V(t,x_{tf}) \leq \beta \sup_{-\tau' \leq s \leq 0}\|\varphi(s)\|^2 e^{-\varepsilon t}. \qquad (65)$$

Since $\mathrm{rank}(E_f) = 2q \leq 2n$,, there exist two nonsingular matrices M and N such that

$$\bar{E} = ME_fN = \begin{bmatrix} I_{2q} & 0 \\ 0 & 0\end{bmatrix}.$$

By Schur complements, it is easy to see that (21) implies (66), shown at the bottom of the page. Define

$$\begin{aligned}
\widetilde{A}(t) &= M\left(A_{af}+\Delta A_{af}(t)\right)N = \begin{bmatrix} \tilde{A}_{11}(t) & \tilde{A}_{12}(t) \\ \tilde{A}_{21}(t) & \tilde{A}_{22}(t)\end{bmatrix} \\
\widetilde{A}_\tau(t) &= M\left(A_{\tau f}+\Delta A_{\tau f}(t)\right)N = \begin{bmatrix} \tilde{A}_{\tau 11}(t) & \tilde{A}_{\tau 12}(t) \\ \tilde{A}_{\tau 21}(t) & \tilde{A}_{\tau 22}(t)\end{bmatrix} \\
\widetilde{A}_h(t) &= M\left(A_{hf}+\Delta A_{hf}(t)\right)N = \begin{bmatrix} \tilde{A}_{h11}(t) & \tilde{A}_{h12}(t) \\ \tilde{A}_{h21}(t) & \tilde{A}_{h22}(t)\end{bmatrix} \\
\bar{P} &= N^TPM^{-1} \\
\bar{Q} &= N^TQN = \begin{bmatrix} Q_1 & Q_0 \\ Q_0^T & Q_2\end{bmatrix} \\
\bar{T} &= N^TTN = \begin{bmatrix} T_1 & T_0 \\ T_0^T & T_2\end{bmatrix}.
\end{aligned} \qquad (67)$$

Combining (20), (66), and (67), we can show that $\widetilde{A}(t)$, $\widetilde{A}_\tau(t)$, $\widetilde{A}_h(t)$, $\bar{P}$, $\bar{Q}$, and $\bar{T}$ satisfy

$$\bar{P}\bar{E} = \bar{E}^T\bar{P}^T \geq 0 \qquad (68)$$

$$\begin{bmatrix} \left(A_{af}+\Delta A_{af}(t)\right)^T P^T + P\left(A_{af}+\Delta A_{af}(t)\right) + Q + T & P\left(A_{\tau f}+\Delta A_{\tau f}(t)\right) & hP\left(A_{hf}+\Delta A_{hf}(s)\right) \\ * & -aQ & 0 \\ * & * & -(1-a)Q \end{bmatrix} < 0. \qquad (66)$$

and

$$\begin{bmatrix} \Psi_2 & \bar{P}\tilde{A}_\tau(t) & h\bar{P}\tilde{A}_h(s) \\ * & -a\bar{Q} & 0 \\ * & * & -(1-a)\bar{Q} \end{bmatrix} < 0 \tag{69}$$

where $\Psi_2 = \tilde{A}^T(t)\bar{P}^T + \bar{P}\tilde{A}(t) + \bar{Q} + \bar{T}$. Obviously, $\bar{P}$ is of the form $\bar{P} = \begin{bmatrix} P_{11} & P_{12} \\ 0 & P_{22} \end{bmatrix}$ and $P_{11} = P_{11}^T > 0$. Substituting $\bar{P}$ into (69) yields (70), shown at the bottom of the page, where

$$\Psi_3 = \tilde{A}_{11}^T(t)P_{11} + P_{11}\tilde{A}_{11}(t) + \tilde{A}_{21}^T(t)P_{12}^T + P_{12}\tilde{A}_{21}(t) + Q_1 + T_1$$

which implies

$$\begin{bmatrix} \Psi_4 & P_{22}\tilde{A}_{\tau 22}(t) & hP_{22}\tilde{A}_{h22}(s) \\ * & -aQ_2 & 0 \\ * & * & -(1-a)Q_2 \end{bmatrix} < 0 \tag{71}$$

where $\Psi_4 = \tilde{A}_{22}^T(t)P_{22}^T + P_{22}\tilde{A}_{22}(t) + Q_2 + T_2$. By Lemma 1, (71) implies that $\tilde{A}_{22}(t)$ and P_{22} are invertible, and a constant $\alpha_1 > 0$ exists such that $\|\tilde{A}_{22}^{-1}(t)\| \le \alpha_1$. Therefore, it follows from [5] and Definition 1 that system (11) is regular and impulse free.

Under a state transformation

$$y_f(t) = N^{-1}x_f(t) = \begin{bmatrix} y_{1f}(t) \\ y_{2f}(t) \end{bmatrix} \tag{72}$$

and noting the structure of $\bar{P}$, we can obtain from (65) that

$$\|y_{1f}(t)\|^2 \le \beta\lambda_{\min}^{-1}(P_{11}) \sup_{-\tau' \le s \le 0} \|\varphi(s)\|^2 e^{-\varepsilon t}. \tag{73}$$

Furthermore, the state transformation $y_f(t) = N^{-1}x_f(t)$ can also lead to the following decomposition of system (11):

$$\begin{aligned} \dot{y}_{1f}(t) = {} & \tilde{A}_{11}(t)y_{1f}(t) + \tilde{A}_{12}(t)y_{2f}(t) \\ & + \tilde{A}_{\tau 11}(t)y_{1f}(t-\tau) + \tilde{A}_{\tau 12}(t)y_{2f}(t-\tau) \\ & + \int_{t-h}^{t} \tilde{A}_{h11}(s)y_{1f}(s)ds \\ & + \int_{t-h}^{t} \tilde{A}_{h12}(s)y_{2f}(s)ds \end{aligned} \tag{74}$$

$$\begin{aligned} 0 = {} & \tilde{A}_{21}(t)y_{1f}(t) + \tilde{A}_{22}(t)y_{2f}(t) \\ & + \tilde{A}_{\tau 21}(t)y_{1f}(t-\tau) + \tilde{A}_{\tau 22}(t)y_{2f}(t-\tau) \\ & + \int_{t-h}^{t} \tilde{A}_{h21}(s)y_{1f}(s)ds \\ & + \int_{t-h}^{t} \tilde{A}_{h22}(s)y_{2f}(s)ds. \end{aligned} \tag{75}$$

Define

$$\begin{aligned} e_f(t) = {} & \tilde{A}_{21}(t)y_{1f}(t) \\ & + \tilde{A}_{\tau 21}(t)y_{1f}(t-\tau) + \int_{t-h}^{t} \tilde{A}_{h21}(s)y_{1f}(s)ds. \end{aligned} \tag{76}$$

From the definition of $\tilde{A}_{21}(t)$, $\tilde{A}_{\tau 21}(t)$, and $\tilde{A}_{h21}(t)$, a scalar $\rho > 0$ exists such that

$$\left\|\tilde{A}_{21}(t)\right\|, \quad \left\|\tilde{A}_{\tau 21}(t)\right\|, \quad \left\|\tilde{A}_{h21}(t)\right\| \le \rho.$$

Then, from (73) and (76), we have

$$\begin{aligned} \|e_f(t)\|^2 \le {} & 3\rho^2\beta\lambda_{\min}^{-1}(P_{11}) \\ & \times \left(1 + e^{\varepsilon\tau'} + h^2 e^{\varepsilon\tau'}\right) \sup_{-\tau' \le s \le 0} \|\varphi(s)\|^2 e^{-\varepsilon t}. \end{aligned} \tag{77}$$

To study the exponential stability of $y_{2f}(t)$, we construct a function as

$$\begin{aligned} J(t) = {} & y_{2f}^T(t)Q_2y_{2f}(t) - ay_{2f}^T(t-\tau)Q_2y_{2f}(t-\tau) \\ & - \frac{(1-a)}{h}\int_{t-h}^{t} y_{2f}^T(s)Q_2y_{2f}(s)ds. \end{aligned} \tag{78}$$

From (75), we obtain, by premultiplying $2y_{2f}^T(t)P_{22}$, that

$$\begin{aligned} 0 = {} & 2y_{2f}^T(t)P_{22}\tilde{A}_{22}(t)y_{2f}(t) + 2y_{2f}^T(t)P_{22}\tilde{A}_{\tau 22}(t)y_{2f}(t-\tau) \\ & + \int_{t-h}^{t} 2y_{2f}^T(t)P_{22}\tilde{A}_{h22}(s)y_{2f}(s)ds + 2y_{2f}^T(t)P_{22}e_f(t). \end{aligned} \tag{79}$$

$$\begin{bmatrix} \Psi_3 & \begin{matrix} P_{11}\tilde{A}_{12}(t) + P_{12}\tilde{A}_{22}(t) \\ +\tilde{A}_{21}^T(t)P_{22}^T + Q_0 + T_0 \end{matrix} & \begin{matrix} P_{11}\tilde{A}_{\tau 11}(t) \\ +P_{12}\tilde{A}_{\tau 21}(t) \end{matrix} & \begin{matrix} P_{11}\tilde{A}_{\tau 12}(t) \\ +P_{12}\tilde{A}_{\tau 22}(t) \end{matrix} & \begin{matrix} hP_{11}\tilde{A}_{h11}(s) \\ +hP_{12}\tilde{A}_{h21}(s) \end{matrix} & \begin{matrix} hP_{11}\tilde{A}_{h12}(s) \\ +hP_{12}\tilde{A}_{h22}(s) \end{matrix} \\ * & \begin{matrix} \tilde{A}_{22}^T(t)P_{22}^T + P_{22}\tilde{A}_{22}(t) \\ +Q_2 + T_2 \end{matrix} & P_{22}\tilde{A}_{\tau 21}(t) & P_{22}\tilde{A}_{\tau 22}(t) & hP_{22}\tilde{A}_{h21}(s) & hP_{22}\tilde{A}_{h22}(s) \\ * & * & -aQ_1 & -aQ_0 & 0 & 0 \\ * & * & * & -aQ_2 & 0 & 0 \\ * & * & * & * & -(1-a)Q_1 & -(1-a)Q_0 \\ * & * & * & * & * & -(1-a)Q_2 \end{bmatrix} < 0 \tag{70}$$

$$J(t) \le -y_{2f}^T(t)[T_2 - \eta_1 I]y_{2f}(t) + \frac{3\rho^2\beta\lambda_{\min}^{-1}(P_{11})\lambda_{\max}^2(P_{22})\left(1 + e^{\varepsilon\tau'} + h^2 e^{\varepsilon\tau'}\right)\sup_{-\tau'\le s\le 0}\|\varphi(s)\|^2}{\eta_1}e^{-\varepsilon t}. \quad (83)$$

$$\zeta_1 = \frac{1}{1+\eta_2}, \quad \zeta_2 = \frac{3\rho^2\beta\lambda_{\min}^{-1}(P_{11})\lambda_{\max}^2(P_{22})\left(1 + e^{\varepsilon\tau'} + h^2 e^{\varepsilon\tau'}\right)\sup_{-\tau'\le s\le 0}\|\varphi(s)\|^2}{\eta_1(1+\eta_2)}.$$

Substituting (79) into (78) and using Lemma 3, we have

$$\begin{aligned} J(t) =& y_{2f}^T(t)\left[\tilde{A}_{22}^T(t)P_{22}^T + P_{22}\tilde{A}_{22}(t) + Q_2\right]y_{2f}(t) \\ &+ 2y_{2f}^T(t)P_{22}\tilde{A}_{\tau 22}(t)y_{2f}(t-\tau) \\ &+ \int_{t-h}^{t} 2y_{2f}^T(t)P_{22}\tilde{A}_{h22}(s)y_{2f}(s)ds \\ &- ay_2^T(t-\tau)Q_2y_{2f}(t-\tau) - \frac{(1-a)}{h} \\ &\times \int_{t-h}^{t} y_{2f}^T(s)Q_2y_{2f}(s)ds + 2y_{2f}^T(t)P_{22}e_f(t) \\ \le& \frac{1}{h}\int_{t-h}^{t}\left[y_{2f}^T(t) \quad y_2^T(t-\tau)\right]U''(t,s)\begin{bmatrix} y_{2f}(t) \\ y_{2f}(t-\tau)\end{bmatrix}ds \\ &+ \eta_1 y_{2f}^T(t)y_{2f}(t) + \frac{1}{\eta_1}e_f^T(t)P_{22}^2e_f(t) \end{aligned} \quad (80)$$

where

$$U''(t,s) = \begin{bmatrix} \tilde{A}_{22}^T(t)P_{22}^T + P_{22}\tilde{A}_{22}(t) + Q_2 \\ +P_{22}\tilde{A}_{h22}(s)\frac{h^2}{(1-a)}Q_2^{-1}\tilde{A}_{h22}^T(s)P_{22}^T & P_{22}\tilde{A}_{\tau 22}(t) \\ * & -aQ_2 \end{bmatrix} \quad (81)$$

and η_1 is any positive scalar.

From (71) and (81) and using Schur complements, we can show that

$$U''(t,s) \le -\mathrm{diag}(T_2 \quad 0). \quad (82)$$

Combining (77), (80), and (82), we obtain (83), shown at the top of the page. Since η_1 can be chosen arbitrarily, η_1 can be thus chosen small enough such that

$$T_2 - \eta_1 I > 0. \quad (84)$$

If η_1 is fixed such that (84) holds, then another constant $\eta_2 > 0$ can be found such that

$$Q_2 + T_2 - \eta_1 I \ge (1+\eta_2)Q_2. \quad (85)$$

Define the second equation at the top of the page. Then, combining (78), (83), and (85), we obtain

$$\begin{aligned} y_{2f}^T(t)Q_2y_{2f}(t) \le& \zeta_1 a y_2^T(t-\tau)Q_2y_{2f}(t-\tau) \\ &+ \frac{\zeta_1(1-a)}{h}\int_{t-h}^{t} y_{2f}^T(s)Q_2y_{2f}(s)ds + \zeta_2 e^{-\varepsilon t}. \end{aligned} \quad (86)$$

Let $f(t) = y_{2f}^T(t)Q_2y_{2f}(t)$. From (86), we have

$$f(t) \le \zeta_1 \sup_{t-\tau'\le s\le t} f(s) + \zeta_2 e^{-\varepsilon t}. \quad (87)$$

Using Lemma 2, one obtains

$$f(t) \le \sup_{-\tau'\le s\le 0} f(s)e^{-\xi_0 t} + \frac{\zeta_2 e^{-\xi_0 t}}{1 - \zeta_1 e^{\xi_0\tau'}}, \quad t \ge 0$$

where $\xi_0 = \min\{\varepsilon, \xi\}$, and $0 < \xi < -(1/\tau')\ln\zeta_1$. Therefore

$$\begin{aligned} \|y_{2f}(t)\|^2 \le& \lambda_{\min}^{-1}(Q_2)\lambda_{\max}(Q_2) \\ &\times \sup_{-\tau'\le s\le 0}\|\varphi_2(s)\|^2 e^{-\xi_0 t} + \frac{\zeta_2\lambda_{\min}^{-1}(Q_2)e^{-\xi_0 t}}{1-\zeta_1 e^{\xi_0\tau'}}, \quad t \ge 0 \end{aligned}$$

which implies by combining (73) and $y_f(t) = N^{-1}x_f(t)$ that $x_f(t)$ is exponentially stable. □

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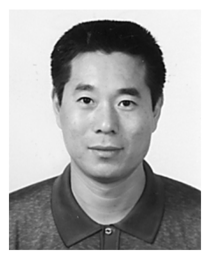

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